Inclusive Access to a Diploma: Reimagining Proficiency for Students with Disabilities

Mathematics I

Systems of Equations

Option #1 Performance Task | Teacher Document

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Inclusive Access to a Diploma: Reimagining Proficiency for Students with Disabilities Initiative Overview

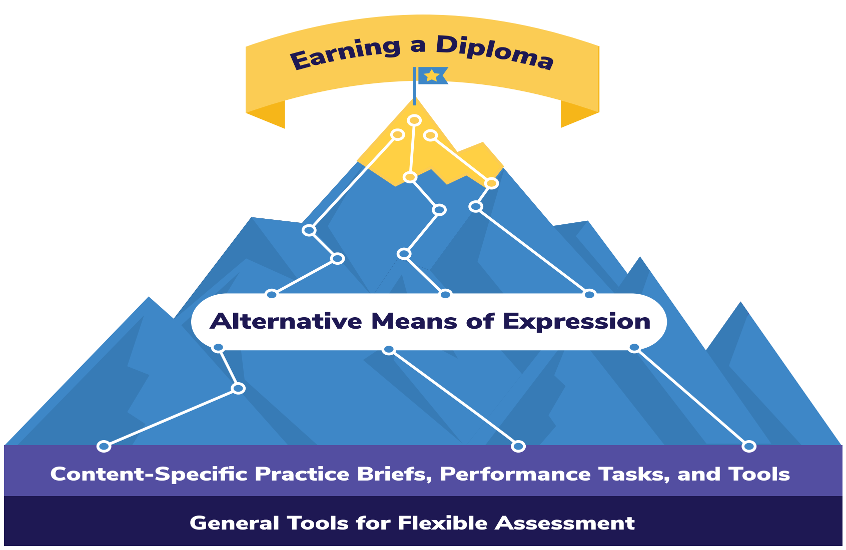
Thank you for utilizing the materials developed for the *Inclusive Access to a Diploma: Reimagining Proficiency for Students with Disabilities* initiative. The strategy and materials developed for this performance task were created through a partnership between the California State Board of Education (SBE), the California Department of Education (CDE), and WestEd. The included performance task is one of many resources developed for this initiative. Senate Bill 101 provided funding to the development of these materials which focus strategically on providing students with disabilities options that would support their high school coursework completion. While the resources are aimed at supporting students with disabilities, LEA governing boards may consider adopting this initiative for all student demographics through Education Code 51225.3. Additionally, because this performance task is based on the general requirements for graduation, it should not be seen as a modification to coursework and is therefore able to be made available to all students with disabilities.[1]

[1] This performance task may not be appropriate for some students with disabilities based on their needs. It is an IEP team decision whether students with disabilities are working toward coursework requirements tied to this initiative and congruent with a standard diploma that meets federal definitions, an alternate pathway to a diploma, or a certificate of completion.

Multiple Routes to Proficiency

Figure 1 provides the conceptual framework for the *Inclusive Access to a Diploma: Reimagining Proficiency for Students with Disabilities* initiative. The graphic displays a mountain with the peek representing coursework requirements for earning a diploma, and the routes up the mountain representing different alternative means of expression made available to students for showing their understanding. The first bar at the foundation of the mountain represents the documents, materials, and resources—such as content-specific practice briefs, performance tasks, and assessment tools—serving as sample alternative means of expression. The second bar at the foundation of the mountain references the general tools, such as the Best Practice Guide, created through this initiative to support schools and districts in providing flexible assessment models.

Figure 1. Conceptual Framework for Inclusive Access to a Diploma Initiative



Overview of the Performance Task

This performance task evaluates students' understanding of key concepts within the Mathematics I Systems of Equations Big Idea. It is divided into parts, each targeting a specific component of the Big Ideas. Each part offers accessible strategies and examples of how students can demonstrate proficiency with the concepts. Various tools, mediums, and connections are provided for teachers to customize the task to the unique needs, cultures, interests, and abilities of their students, promoting an inclusive and relevant educational experience.

When preparing this performance task, distinguish between the flexible and fixed elements to ensure students have multiple ways to demonstrate their knowledge without compromising the concepts' depth and rigor within the standards. Furthermore, educators should always consult a student's Individualized Education Program (IEP) to ensure that all required accommodations and supplementary aids are provided during the assessment.

Additional information on providing alternative means of expression can be found in the best practice guides and content-aligned practice briefs defined as part of the California Alternative Means to a Diploma Project.

Administering the Performance Task and Embedding Resources for Students

Each part of this task is broken into a series of items for administration. This section provides guidance to the educator on how to administer each part of the task and support the student in demonstrating their understanding of the Big Idea. As you plan to administer this performance task, it is suggested to review these recommendations as they offer associated key vocabulary, appropriate and inappropriate resources, and potential methods and means of expression.

Key Vocabulary Associated with the Standards

Understanding and correctly using the key vocabulary terms below is essential to demonstrating proficiency with the concepts that fall under these Big Ideas; thus, unless otherwise noted, this vocabulary cannot be taught during completion of the task. However, students may still use tools and resources embedded in the environment (such as word walls, notebooks, word banks, glossary, and so forth) for support with these words.

- system, equation, system of equations, expression, graph, input, output, variable, dependent variable, independent variable, point, coordinates, value, evaluate, substitute, intersect, point of intersection, linear, linear equation, *x*-axis, *y*-axis, *y*-intercept, slope, rate of change, unit rate, common difference, interval, represent / representation, solution, solution set, product, sum, ordered pair

Strategies for Supporting Students

The following sections describe appropriate and inappropriate resources to provide students as they complete a task.

Appropriate Resources

Appropriate resources maintain the rigor of the standards while also accommodating any student difficulties such as confusion or anxiety or providing a resource the student could use to complete the task.

- reading the item to the student
- answering clarifying questions vocabulary (for example, “create a function” means to find or come up with a function or “possible combinations” means “all the different ways she can make a necklace;” or “What does the model suggest about the relationship” means “what does the model tell you about the relationship”)
- helping the student to make sense of the item by asking questions such as, “What is this question asking you to figure out? What important information does the question give you? Are there any words you want to ask about or look up?”
- separating out each item across multiple class periods or providing frequent breaks within a class period to complete the full task
- prompting the student to describe the problem and constraints in their own words
- reviewing a student’s response and pointing out where they need to give more detail or elaborate on their reasoning, or where they need to be more mathematically precise
- offering drawing tools (paper/pencil, colored pencils, straight edge, or computer drawing/graphing technology)
- supporting the student in accessing any technology tools, such as Desmos or GeoGebra
- helping the student to access resources to remind them of the meaning of mathematical terms such as “system of equations” or “system of inequalities”

Inappropriate Resources

The list of inappropriate resources below identifies what assistance should be avoided, as these supports interfere with the student's independent completion of the task and may compromise its utility in terms of assessing the student's proficiency with the standards:

- explaining or re-teaching mathematical concepts
- demonstrating to students how to solve similar problems so that the student can imitate the teacher's strategy
- explaining to the student step-by-step how to complete the items
- breaking the problems down into a step-by-step process[2]

Potential Alternative Means of Expression

Potential methods and means of expression show the various ways students can demonstrate their knowledge of the standards being assessed in this part of the task.

Students can explain their reasoning by

- using paper and pen or pencil
- typing a response using a word processor
- using speech-to-text software
- explaining in words to the teacher
- dictating to a scribe[3]

Students can graph systems in the following ways:

- using paper and pencil
- using a handheld graphing calculator
- using graphing software such as Desmos

[2] Part of what this task is assessing is the student's ability to read a contextual problem and formulate a mathematical strategy for solving it.

[3] In this situation, it is important for the scribe to be careful to record **only** what the student explicitly communicates, rather than making interpretations and "filling in the blanks" based on what they think the student meant.

PART 1. Spirit Bracelets

Investigating Systems of Equations

Part 1 of this performance task includes

- associated standards that will be assessed
- rubrics that assess each item
- the student task requirements
- sample student responses

Teachers should familiarize themselves with the related standards, review the student task, analyze each item's rubric, and view the sample student responses to sufficiently prepare for using the task to assess students' proficiency with the standards addressed in the task. Additionally, teachers must be careful to incorporate any IEP-defined supplementary aids and services specific to individual students with disabilities taking this performance task.

Task Alignment to Key Elements of Big Ideas and Standards

Clusters of content standards exist within the Big Ideas, allowing the Big Ideas to demonstrate the central concepts and key understanding of the course content. The indicator statements provide the teacher with the key concepts being evaluated in each Big Idea as well as the associated content standards centered within the Big Idea of this task. The indicator statements come from the 2023 *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve* and are aligned to the California-adopted mathematics state standards. Each part of the performance task will target a different indicator or standard of the Big Idea. Please review the outline below to see how the entirety of the Big Idea is assessed across all the parts of the performance task.

Systems of Equations: Big Idea Indicator 1

Students investigate real situations that include data for which systems of one or two equations or inequalities are helpful, paying attention to units. Investigations include linear, quadratic, and absolute value [relationships].

- **Solve systems of equations.** [Linear systems]
  - *(Item 1)* **A-REI.6** Solve systems of linear equations exactly and approximately (for example, with graphs), focusing on pairs of linear equations in two variables.
- **Represent and solve equations and inequalities graphically.** [Linear and exponential; learn as general principle]
  - *(Item 1)* **A-REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Systems of Equations: Big Idea Indicator 2

Students use technology tools strategically to find [systems'] solutions and approximate solutions, [construct] viable arguments, [interpret] the meaning of the results, and [communicate] them in multidimensional ways.

- **Represent and solve equations and inequalities graphically.**
  - *(Item 1)* **A-REI.11** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, for example, using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Part 1. Item

Deborah is making spirit bracelets to sell at the fundraiser for a school club. She wants to sell as many bracelets as possible and raise as much money as possible.

Item 1. Directions [Student Document, p. 2]

Item 1 has one task. Complete the task below.

Item 1 Task

Deborah's club decided they would sell two different types of spirit bracelets—**basic** Spirit Bracelets and **ornate** Spirit Bracelets. They decided to sell the basic bracelets for $5 each and the ornate bracelets for $8 each. Their fundraising goal is to make more than $5,000. A local craft store has donated enough materials to make up to 800 bracelets in total, but the club needs to decide how many of each type to make.

Deborah writes the following system of equations to represent the situation above, where x represents the number of basic bracelets made and y represents the number of ornate bracelets made:

$$\begin{Bmatrix} 5x+8y=5{,}000 \\ x+y=800 \end{Bmatrix}$$

Show and explain at least **TWO** different ways that the club could use a system of inequalities to determine the number of each type of bracelet to make and explore how much money they could potentially earn by doing so.

One of your methods should use technology such as a graphing calculator or computer algebra software, one should include a graphical representation, and one should use algebraic equations (**A-REI.6**, **A-REI.10**, **A-REI.11**).

A Rubric for Assessing Item 1

A-REI.6 Solve systems of linear equations exactly and approximately (for example, with graphs), focusing on pairs of linear equations in two variables.

A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, for example, using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Rubric for Part 1, Item 1

Attempted	Approaching	Proficient
The student attempts to describe the feasible region or a strategy for determining it, but there are multiple major conceptual errors. —OR— The explanation is missing or so unclear and imprecise that it is not possible to determine the student's level of conceptual understanding.	The student presents **one** correct strategy with a coherent explanation of the strategy and no major conceptual errors. The explanation may lack clarity, specificity, or precise language regarding the connections between the solution set of the system, the point of intersection of the lines $x + y = 800$ and $5x + 8y = 5{,}000$, and the combinations of ornate and basic bracelets that meet both constraints, but overall, the response is generally conceptually correct. The student may include 2–3 minor mistakes due to transcription or calculation errors —OR— the student presents **two** correct strategies with coherent explanations. The student may include one major conceptual error, but overall, the response is *generally* conceptually correct. The explanation may lack clarity, specificity, or precise language regarding the connections between the solution set of the system, the point of intersection of the lines	The student presents **two** correct strategies for determining the options for many of each type of bracelet to make, with no major conceptual errors. One of the strategies ● uses technology (such as a graphing calculator or online graphing software), ● uses graphical techniques to approximately determine and describe the solution to the equation, ● uses algebraic techniques to find the exact solution to the system $\left(466\frac{2}{3}, 333\frac{1}{3}\right)$, or ● includes a clear and coherent explanation of each strategy. The graphical strategy ● includes a graph that shows both inequalities correctly plotted and shaded; ● correctly labels the axes, lines, scale, and vertices of the feasible region; ● explains that the graph of an inequality in two variables is the set of all its solutions plotted or shaded in the coordinate plane; or ● identifies the *overlapping* plotted points and shaded areas as the solution to the *system* of inequalities. The algebraic strategy

Attempted	Approaching	Proficient
	$x + y = 800$ and $5x + 8y = 5{,}000$, and the combinations of ornate and basic bracelets that meet both constraints. The student may include 2–3 minor mistakes due to transcription or calculation errors.	● demonstrates and explains a valid algebraic strategy for identifying the solution set of the system of inequalities; ● correctly interprets the point of intersection of the lines $x + y = 800$ and $5x + 8y = 5{,}000$ $\left(466\frac{2}{3}, 333\frac{1}{3}\right)$ as one of the vertices of the triangle that bounds the feasible region; or ● correctly interprets the solution set of the system in context (that is, the coordinates of any point in the feasible region correspond to a combination of ornate and basic bracelets that satisfies the constraints).

Part 1. Sample Student Response

The content below provides a sample of proficient responses from a student. The text that leads with “Student Voice” is an example of how a student might respond to each item. This section should only serve as a model—different students will arrive at solutions in different ways.

Item 1 [Student Document, p. 2]

Deborah’s club decided they would sell two different types of spirit bracelets—**basic** Spirit Bracelets and **ornate** Spirit Bracelets. They decided to sell the basic bracelets for $5 each and the ornate bracelets for $8 each. Their fundraising goal is to make more than $5,000. A local craft store has donated enough materials to make up to 800 bracelets in total, but the club needs to decide how many of each type to make.

Deborah writes the following system of equations to represent the situation above, where *x* represents the number of basic bracelets made and *y* represents the number of ornate bracelets made:

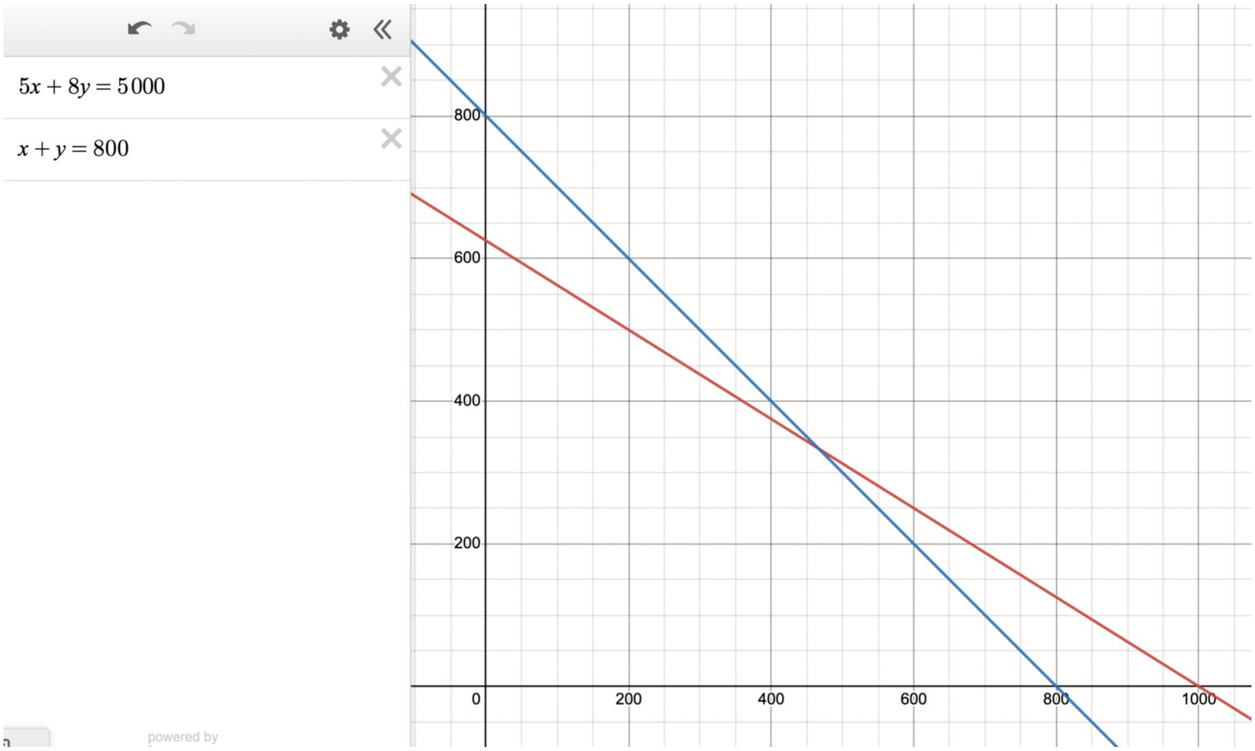
$$\begin{cases} 5x+8y=5{,}000 \\ x+y=800 \end{cases}$$

Show and explain at least **TWO** different ways that the club could use a system of inequalities to determine the number of each type of bracelet to make and explore how much money they could potentially earn by doing so.

One of your methods should use technology such as a graphing calculator or computer algebra software, one should include a graphical representation, and one should use algebraic equations.

Student Voice: One way to figure out how many of each type of bracelet they should make is to graph the two inequalities:

Figure 2. Sample Student-Generated Graph for Item 1



Student Voice: In this graph, the blue line represents all the (*x*, *y*) points that are combinations where the club makes $5,000. The red line represents all the (*x*, *y*) points that are combinations that add up to 800 bracelets. We can see that there is only one point that appears on both lines, that is, only one point where the group has made 800 bracelets and also made $5,000—and that is the point where the lines intersect. From looking at the graph, it seems like that point is about (475, 325), meaning they make 475 basic bracelets and 325 ornate.

A second way, if we want to find the exact coordinates of the intersection point, is to use algebra to find the solution of the system $\begin{cases} 5x + 8y = 5{,}000 \\ x + y = 800 \end{cases}$.

One way to solve the system is to multiply the second equation by -8 on both sides to get

$$\begin{cases} 5x + 8y = 5{,}000 \\ -8x + (-8)y = -8 \cdot 800 \end{cases}$$

$$\begin{cases} 5x + 8y = 5{,}000 \\ -8x - 8y = -6{,}400 \end{cases}$$

Then add the two together to get $-3x=-1{,}400$. This shows us that $x=\frac{-1{,}400}{-3}=466\frac{2}{3}$. Since $x+y=800$, that means $y=800-466\frac{2}{3}=333\frac{1}{3}$. So, the exact coordinates of vertex C are $\left(466\frac{2}{3},333\frac{1}{3}\right)$, which are pretty close to our eyeball estimate of (475, 325). Using this method, the club might decide that they should make 466 or 467 basic bracelets and 333 or 334 ornate bracelets.

PART 2. Proving A Property of Systems

Type III Operations

Part 2 of this performance task includes

- associated standards that will be assessed
- rubrics that assess each item
- the student task requirements
- sample student responses

Teachers should familiarize themselves with the related standards, review the student task, analyze each item's rubric, and view the sample student responses to sufficiently prepare for using the task to assess students' proficiency with the standards addressed in the task. Additionally, teachers must be careful to incorporate any IEP-defined supplementary aids and services specific to individual students with disabilities taking this performance task.

Task Alignment to Key Elements of Big Ideas and Standards

Clusters of content standards exist within the Big Ideas, allowing the Big Ideas to demonstrate the central concepts and key understanding of the course content. The indicator statements provide the teacher with the key concepts being evaluated in each Big Idea as well as the associated content standards centered within the Big Idea of this task. The indicator statements come from the *Mathematics Framework* and are aligned with the California-adopted mathematics state standards. Each part of the performance task will target a different indicator or standard related to the Big Idea. Please review the outline below to see how the entirety of the Big Idea is assessed across all the parts of the performance task.

Systems of Equations: Big Idea Indicator 1

Students investigate real situations that include data for which systems of one or two equations or inequalities are helpful, paying attention to units. Investigations include linear, quadratic, and absolute value [relationships].

- **Solve systems of equations.** [Linear-linear and linear-quadratic]
    - *(Item 1)* **A-REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Systems of Equations: Big Idea Indicator 2

Students use technology tools strategically to find [systems'] solutions and approximate solutions, [construct] viable arguments, [interpret] the meaning of the results, and [communicate] them in multidimensional ways.

- **Represent and solve equations and inequalities graphically.** [Linear and exponential; learn as general principle]
    - *(Item 2)* **A-REI.11** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, for example, using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Part 2. Items

Item 1. Directions [Student Document, p. 3]

Item 1 has one task and builds on content from Part 1. Complete the task below.

Item 1 Task

The following year, Deborah's club created the following system of equations to determine how much to charge for their spirit bracelets (**A-REI.5**).

SYSTEM A:

$$\begin{Bmatrix} 5x+10y=6{,}000 \\ x+y=1{,}000 \end{Bmatrix}$$

Deborah first multiplied both sides of the second equation by 2:

$$\begin{Bmatrix} 5x+10y=6{,}000 \\ 2x+2y=2\cdot 1{,}000 \end{Bmatrix} \rightarrow \begin{Bmatrix} 5x+10y=6{,}000 \\ 2x+2y=2{,}000 \end{Bmatrix}$$

Then she replaced the second equation with the **sum** of the two equations:

$$\begin{matrix} 5x+10y=6{,}000 \\ 5x+10y+2x+2y=6{,}000+2{,}000 \end{matrix}$$

SYSTEM B:

$$\begin{Bmatrix} 5x+10y=6{,}000 \\ 7x+12y=8{,}000 \end{Bmatrix}$$

She says that any time you follow this process, the new system you get will have the **same solution set** as the original system.

Use mathematics tools and strategies to show and explain why this is true.

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A Rubric for Item 1

A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Rubric for Part 2, Item 1

Attempted	Approaching	Proficient
The student attempts to explain why replacing an equation in a system with a multiple of that equation or with the sum of that equation and a multiple of the other gives a system with the same solutions as the original, but there are multiple major conceptual errors. —OR— The explanation is missing or so unclear and imprecise that it is not possible to determine the student's level of conceptual understanding.	The student correctly explains one of the statements under "proficient" with no major conceptual errors. The explanation may lack clarity, specificity, or precise language, but overall, the response is generally conceptually correct. May include 2–3 minor mistakes due to transcription or calculation errors.	The student correctly explains why replacing an equation in a system with a multiple of that equation creates a new system with the same solutions as the original system (for example, by noting that an equation and all its multiples are simultaneous equations or by showing and explaining algebraically that the two must have the same solutions). The student correctly explains why adding one true equation to another true equation creates a third true equation (that is, because adding the same value to both sides of a true equation keeps it in balance).

Item 2. Directions [Student Document, p. 4]

Item 2 is one task and builds on content from Parts 1 and 2. Complete the task below.

Item 2 Task

Fernando likes to solve systems of equations by first rewriting both equations as **linear functions**. He rewrote Deborah's system like this:

$$\begin{Bmatrix} f(x) = 600 - \frac{1}{2}x \\ g(x) = 1{,}000 - x \end{Bmatrix}$$

"Now it is easy to solve. Because we want to find the point where both $f(x)$ and $g(x)$ give the same output for the same input (*x* value), we can find the solution by setting $600 - \frac{1}{2}x$ and $1{,}000 - x$ equal to each other and solving for *x*."

$$600 - \frac{1}{2}x = 1{,}000 - x$$

Explain using a **graphical representation** why Fernando is correct. Then find the approximate solution using any mathematical method that makes sense to you.

A Rubric for Assessing Item 2

A-REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, for example, using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Rubric for Part 2, Item 2

Attempted	Approaching	Proficient
The student attempts to present a graph but does not attempt an explanation, or vice versa —OR— the graph has multiple major errors. Attempts to explain why the x-coordinate of the intersection point is a solution to $f(x) = g(x)$, but there are multiple major conceptual errors. —OR— The explanation is missing or so unclear and imprecise that it is not possible to determine the student's level of conceptual understanding.	The student presents a mostly correct graph of the system; though errors may be present, it is still clear how the graph supports and is connected to the explanation. The explanation regarding why the x-coordinate of the intersection point is a solution to $f(x) = g(x)$ is generally correct but may lack clarity, specificity, or precise language. The student correctly identifies the solution to the system ($x = 800$, $y = 200$) to within a reasonable margin of error, either by identifying the coordinates of the point of intersection on the graph or by using input–output tables. A couple of small, insubstantial errors are okay (that is, transcription or calculation errors). —OR— The student meets all proficient criteria except identifying the correct solution ($x = 800$, $y = 200$).	The student presents a mostly correct graph of the system; though errors may be present, it is still clear how the graph supports and is connected to the explanation. The student uses the graph to correctly explain why the x-coordinate of the point of intersection of $f(x)$ and $g(x)$ is a solution to the equation $f(x) = g(x)$. For example, ● the coordinates of the point of intersection are the (x, y) values that make both equations true—that is, $f(x) = y$ and $g(x) = y$, so $f(x)$ must equal $g(x)$ for that set of values. ● to find a solution for the equation $f(x) = g(x)$, we need to find an x value where $f(x)$ and $g(x)$ give the same output. If there were such a point on the graph, it would appear on the graph of both functions, meaning the functions must intersect at that point. ● an argument similar to the above using specific values (that is, $x = 800$, $y = 200$) as an example. The student correctly identifies the solution to the system ($x = 800$, $y = 200$) either by identifying the coordinates of the point of intersection on the graph or by using input–output tables.

Part 2. Sample Student Response

The content below provides a sample of proficient responses from a student. The text that leads with "Student Voice" is an example of how a student might respond to each item. This section should only serve as a model—different students will arrive at solutions in different ways.

Item 1 [Student Document, p. 3]

The following year, Deborah's club created the following system of equations to determine how much to charge for their spirit bracelets:

SYSTEM A:

$$\begin{Bmatrix} 5x+10y=6{,}000 \\ x+y=1{,}000 \end{Bmatrix}$$

Deborah first multiplied both sides of the second equation by 2:

$$\begin{Bmatrix} 5x+10y=6{,}000 \\ 2x+2y=2\cdot 1{,}000 \end{Bmatrix} \rightarrow \begin{Bmatrix} 5x+10y=6{,}000 \\ 2x+2y=2{,}000 \end{Bmatrix}$$

Then she replaced the second equation with the **sum** of the two equations:

$$5x+10y=6{,}000$$

$$5x+10y+2x+2y=6{,}000+2{,}000$$

SYSTEM B:

$$\begin{Bmatrix} 5x+10y=6{,}000 \\ 7x+12y=8{,}000 \end{Bmatrix}$$

She says that any time you follow this process, the new system you get will have the **same solution set** as the original system. Use mathematics tools and strategies to show and explain why this is true.

Student Voice: Any time we multiply both sides of an equation by the same number, the new equation will have the same solutions as the original. We can see this if we graph an equation and its “multiples”—they all have the same graph, which means that they have the same set of solutions:

Figure 3. Sample Student-Generated Graph for Item 1—Graph A

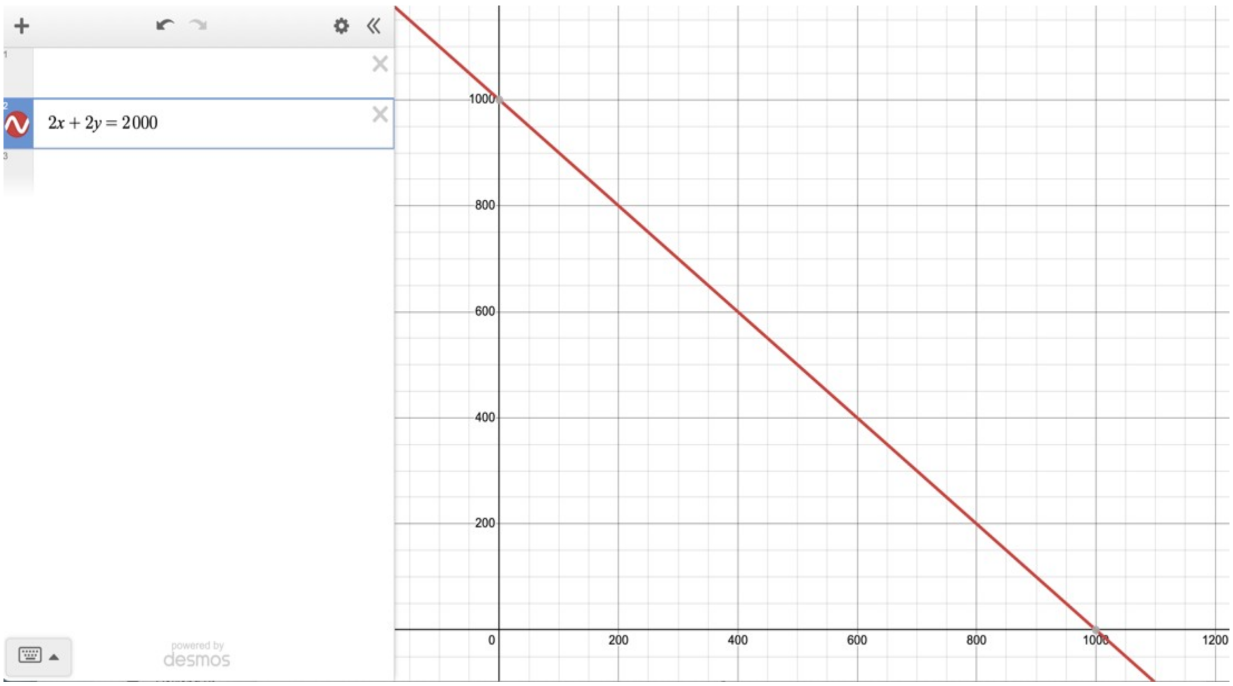


Figure 4. Sample Student-Generated Graph for Item 1—Graph B

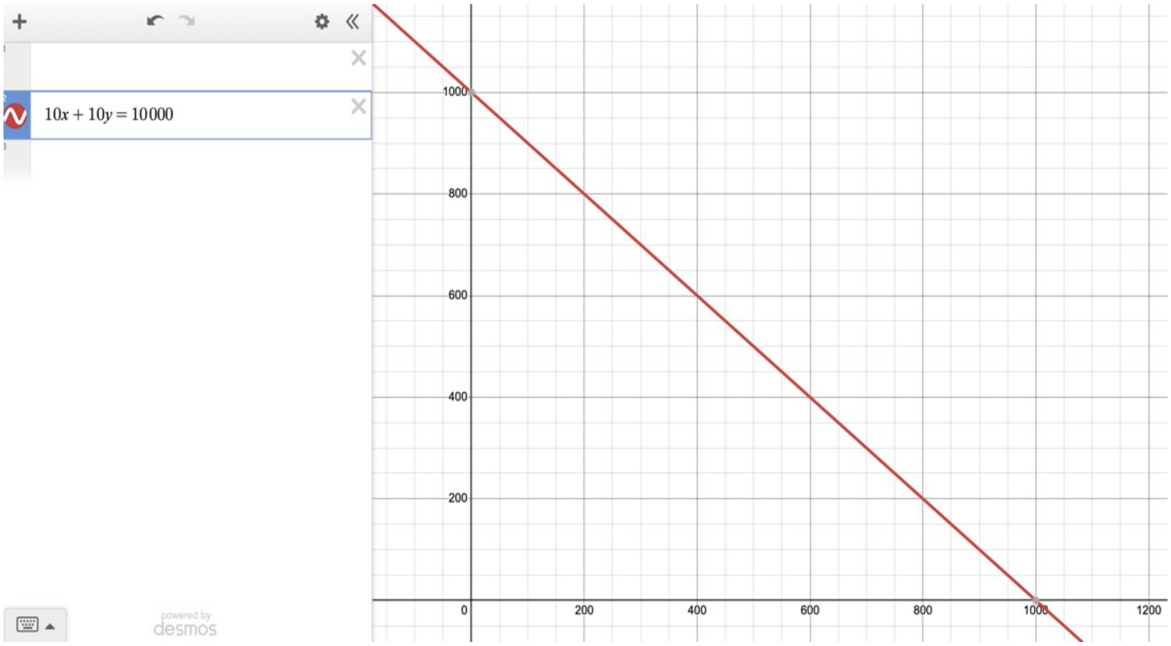
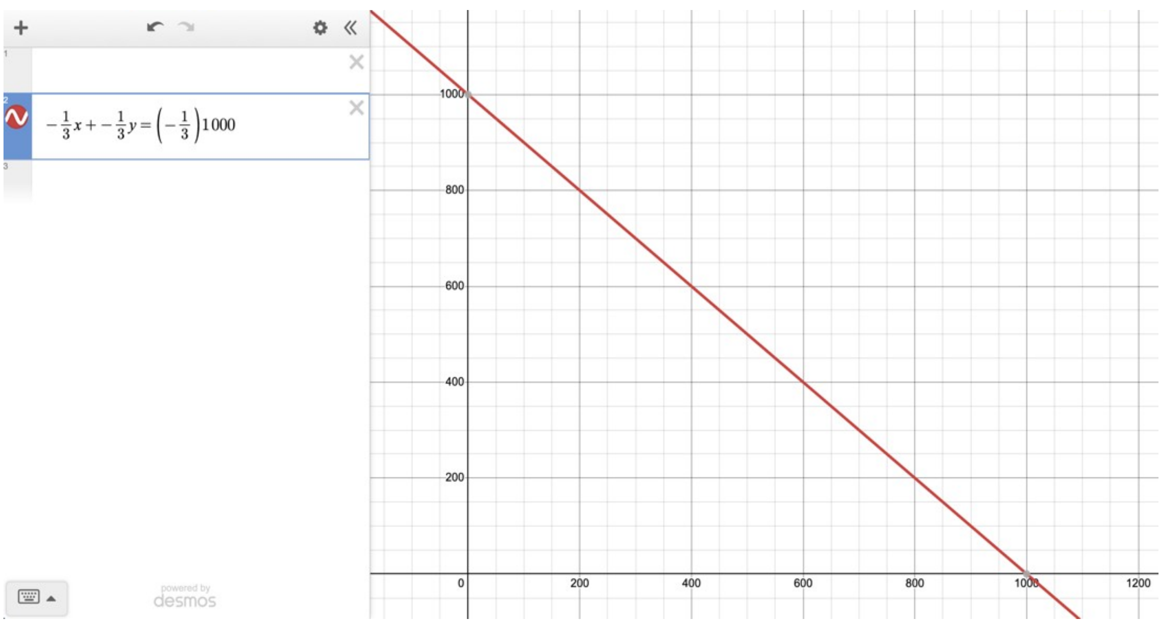


Figure 5. Sample Student-Generated Graph for Item 1—Graph C

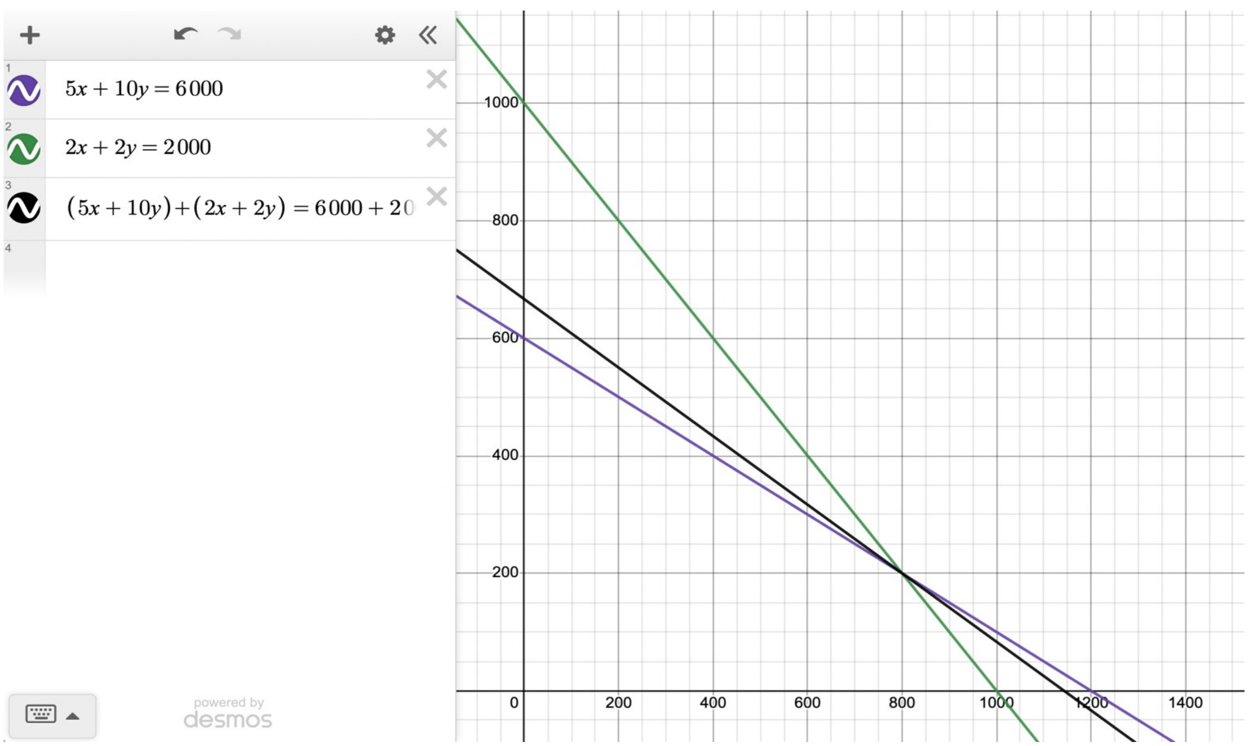


Student Voice: Another way to see this is to suppose that (*a*, *b*) is a solution to an equation like $x + y = 1{,}000$. That is, $a + b = 1{,}000$. If $a + b = 1{,}000$ is a true equation, then $k \cdot a + k \cdot b = k \cdot 1{,}000$ must also be a true equation. So (*a*, *b*) is also a solution to $kx + ky = 1{,}000k$.

So, if you have a system of two equations, and you multiply both sides of one of the equations by any number, you haven't changed the solutions of that equation. That means the solutions to the system cannot have changed, either.

We can also see by looking at graphs that an equation that is the sum of the two equations in the system (the black line) passes through the point of intersection of the two equations in the system (the green and purple lines)—so we could replace either of those equations with this new equation and still have the same point of intersection (which is the solution of the system).

Figure 6. Sample Student Graph for Item 1—Graph D



Student Voice: Another way to see it is to suppose that the point (*a*, *b*) is the solution of this system. So $5a+10b=6{,}000$ and $a+b=1{,}000$. If $5a+10b$ and 6,000 are the same amount, and we add that same amount to each side of $2a+2b=2{,}000$, then the new equation will still be balanced (because we are adding the same amount to both sides).

We would have $(5a+10b)+(2a+2b)=6{,}000+2{,}000$, or $7a+12b=8{,}000$. So, we can see that the point (*a*, *b*) is still a solution for this new equation.

We know that if we multiply one equation by some number to create a new system, that system will still have the same solution as the original one. And if we then add the two equations together and replace one of them with the new equation, it will *also* still have the same solution. Therefore, replacing one equation in the system by the sum of that equation and a multiple of the other will always produce a system with the same solutions as the original system.

Item 2 Task [Student Document, p. 4]

Fernando likes to solve systems of equations by first rewriting both equations as **linear functions**. He rewrote Deborah's system like this:

$$\begin{cases} f(x)=600-\frac{1}{2}x \\ g(x)=1{,}000-x \end{cases}$$

"Now it is easy to solve. Because we want to find the point where both $f(x)$ and $g(x)$ give the same output for the same input (*x* value), we can find the solution by setting $600-\frac{1}{2}x$ and $1{,}000-x$ equal to each other and solving for *x*."

$$600-\frac{1}{2}x=1{,}000-x$$

Explain using a **graphical representation** why Fernando is correct. Then, find the approximate solution using any mathematical method that makes sense to you.

Student Voice: First, let's find the solution to the system by graphing the two functions and seeing where their graphs intersect:

Figure 7. Sample Student-Generated Graph for Item 2

Student Voice: It looks like the solution is *x* = 800, *y* = 200 since the two lines cross at (800, 200). When we substitute *x* = 800 into the first function, we get $f(x)=600-\left(\frac{1}{2}\right)800=600-400=200$, and when we substitute *x* = 800 into the second function, we get $g(x)=1{,}000-800=200$. The point where the two lines intersect is the point where both functions give the same output for the same *x* value. So, if we substitute that *x* value into the equation $f(x)=g(x)$, we will see that we get a true equation. (In this case, 200 = 200.) So, we can see that the *x*-coordinate of the intersection point of $f(x)=g(x)$ is a solution to that equation.

PART 3. Spirit Bracelets Revisited

Applying Systems of Equations

Part 3 of this performance task includes

- associated standards that will be assessed
- rubrics that assess each item
- the student task requirements
- sample student responses

Teachers should familiarize themselves with the related standards, review the student task, analyze each item's rubric, and view the sample student responses to sufficiently prepare for using the task to assess students' proficiency with the standards addressed in the task. Additionally, teachers must be careful to incorporate any IEP-defined supplementary aids and services specific to individual students with disabilities taking this performance task.

Task Alignment to Key Elements of Big Ideas and Standards

Clusters of content standards exist within the Big Ideas, allowing the Big Ideas to demonstrate the central concepts and key understanding of the course content. The indicator statements provide the teacher with the key concepts being evaluated in each Big Idea as well as the associated content standards centered within the Big Idea of this task. The indicator statements come from the *Mathematics Framework* and are aligned with the California-adopted mathematics state standards. Each part of the performance task will target a different category or standard of the Big Idea. Please review the outline below to see how the entirety of the Big Idea is assessed across all the parts of the performance task.

Systems of Equation: Big Idea Indicator 1

Students investigate real situations that include data for which systems of one or two equations or inequalities are helpful, paying attention to units. Investigations include linear, quadratic, and absolute value [relationships].

- **Represent and solve equations and inequalities graphically.** [Linear and exponential; learn as general principle]
    - **A-REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Part 3. Item

Item 1. Directions [Student Document, p. 5]

Item 1 has one task and builds on content from Parts 1, 2, and 3. Complete the task below.

Item 1 Task

One year, the club needs to make $4,500 and has enough materials to make 1,000 bracelets. They decided to charge $5 for **basic** bracelets and $10 for **ornate** bracelets. Deborah represented the two constraints using the following system of equations:

$$\begin{Bmatrix} 4x+8y=6{,}400 \\ x+y=900 \end{Bmatrix}$$

Lian points out to Deborah that the club wants to make *more* than $6,400, not $6,400 exactly. “It is also okay if we sell less than 900 bracelets. It’s just that we can only make *up to* 900 bracelets.” Lian says that this means they actually need a system of *inequalities,* rather than a system of *equations.*

Lian adjusts Deborah’s system of equations so that it is a system of inequalities instead (**A-REI.12**):

$$\begin{cases} 4x+8y>6{,}400 \\ x+y\leq 900 \end{cases}$$

Represent and solve Lian’s system of inequalities graphically.

A Rubric for Assessing Item 1

A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Rubric for Part 3, Item 1

Attempted	Approaching	Proficient
The student attempts to describe a feasible region or a strategy for determining it, but there are multiple major conceptual errors. —OR— The explanation is missing or so unclear and imprecise that it is not possible to determine the student's level of conceptual understanding.	The student presents a mostly correct graph of the systems of inequalities, though there may be one significant error such as ● a line is dotted when it should be solid or vice versa, ● the wrong side of the line is shaded, ● an intercept is clearly in the wrong place, ● the point of intersection is significantly off, ● an axis is not labeled, or ● an axis is missing its scale. The student uses the graph to correctly explain the solution set; the explanation may lack clarity, specificity, or precise language as regards the connections between the solution set of the system, the point of intersection of the lines $x + y = 900$ and $4x + 8y = 6{,}400$, and the overlapping shaded region, but overall, the response is generally conceptually correct. A couple of small, insubstantial errors are okay (that is, transcription or calculation errors) —OR— the student meets all proficient criteria except identifying the correct solution ($x = 800$, $y = 200$).	The student presents a correct graph of the system of inequalities. The student includes a graph that shows both inequalities correctly plotted and shaded. The axes, lines, scales, and vertices of the feasible region are correctly labeled. The student identifies the overlapping plotted points and shaded areas as the solution to the system of inequalities. The student uses the graph to correctly explain the solution set for the system of inequalities (the triangle with vertices (0, 800), (0, 900), and (200, 700), inclusive of the part of the boundary that overlaps with the y-axis and the line $x + y = 900$ but not the line $4x + 8y = 6{,}400$).

Part 3. Sample Student Response

The content below provides a sample of proficient responses from a student. The text that leads with "Student Voice" is an example of how a student might respond to each item. This section should only serve as a model—different students will arrive at solutions in different ways.

Item 1 Task [Student Document, p. 5]

One year, the club needs to make $4,500 and has enough materials to make 1,000 bracelets. They decided to charge $5 for **basic** bracelets and $10 for **ornate** bracelets. Deborah represented the two constraints using the following system of equations:

$$\begin{Bmatrix} 4x + 8y = 6{,}400 \\ x + y = 900 \end{Bmatrix}$$

Lian points out to Deborah that the club wants to make *more* than $6,400, not $6,400 exactly. "It is also okay if we sell less than 900 bracelets. It's just that we can only make *up to* 900 bracelets." Lian says that this means they actually need a system of *inequalities,* rather than a system of *equations.*

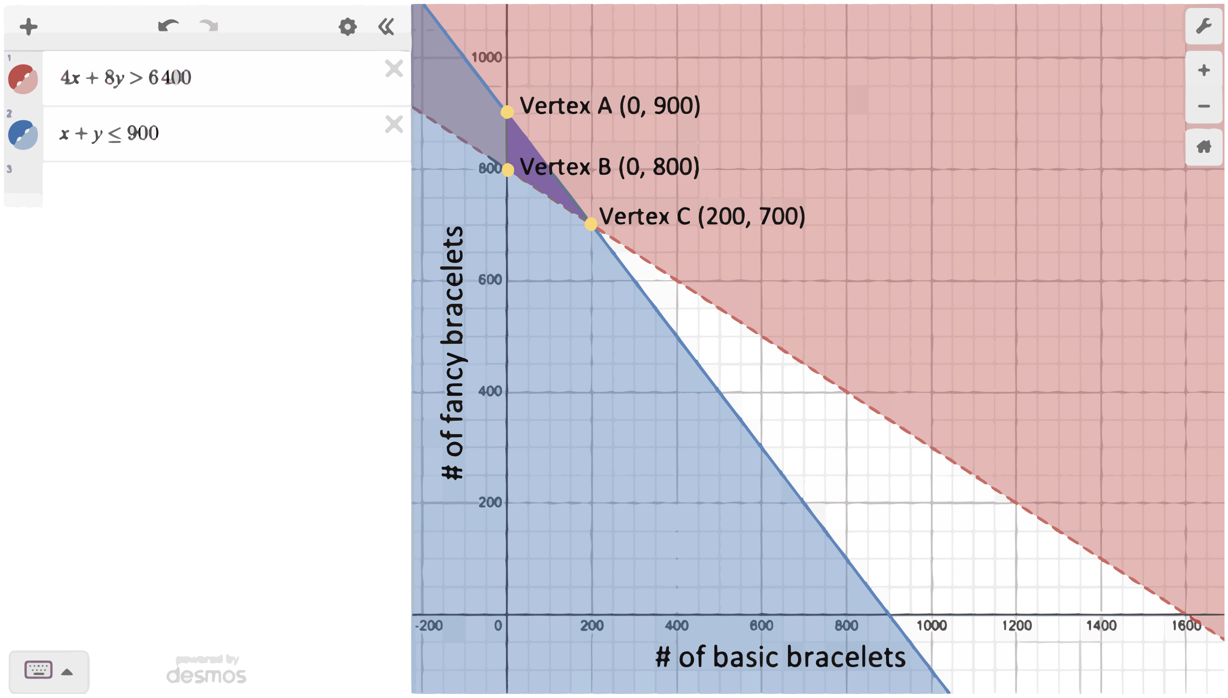
Lian adjusts Deborah's system of equations so that it is a system of inequalities instead:

$$\begin{Bmatrix} 4x + 8y > 6{,}400 \\ x + y \le 900 \end{Bmatrix}$$

Represent and solve Lian's system of inequalities graphically.

Student Voice: First, we need to graph the two inequalities. The shaded blue part of the graph represents all the (x, y) points that are combinations where $4x + 8y$ is over $6,400. (The line is dotted because, at the points on the line, $4x + 8y$ would equal exactly $6,400 but not more, so they are not solutions to the blue inequality.) The shaded red part represents all the (x, y) points where $x + y$ adds up to 900 bracelets or less. (This line is solid because exactly 900 is okay so those points count.) The section where the blue and red shading overlap (purple) are solutions to the system because the points where $4x + 8y$ is over 6,400 ***and*** $x + y$ is 900 or less.

Figure 8. Sample Student-Generated Graph for Item 1



Student Voice: By looking at the graph, we can see that the vertices of the triangular solution set are (0, 800), (0 900), and (200, 700). Any point inside this triangle (or on the part of the red line or *y*-axis that makes up the border of the triangle) is a point where $4x + 8y$ is over 6,400 ***and*** $x + y$ is 900 or less, so it will be a solution to the system. The points on the *y*-axis between (0, 800) and (0, 900) are also part of the solution set, as are the points on the line $x + y = 900$ between (0, 900) and (200, 700), but the points on the line $4x + 8y = 6{,}400$ between (0, 900) and (200, 700) are not part of the solution set.