



Inclusive Access to a Diploma:
Reimagining Proficiency
for Students with Disabilities



Mathematics I – Geometry Supertask Shapes and Structures, Building with Triangles, & Transformations and Congruence

Option #1 Performance Task | Teacher Document

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Inclusive Access to a Diploma: Reimagining Proficiency for Students with Disabilities Initiative Overview

Thank you for utilizing the materials developed for the *Inclusive Access to a Diploma: Reimagining Proficiency for Students with Disabilities* initiative. The strategy and materials developed for this performance task were created through a partnership between the California State Board of Education (SBE), the California Department of Education (CDE), and WestEd. The included performance task is one of many resources developed for this initiative. Senate Bill 101 provided funding to the development of these materials which focus strategically on providing students with disabilities options that would support their high school coursework completion. While the resources are aimed at supporting students with disabilities, LEA governing boards may consider adopting this initiative for all student demographics through Education Code 51225.3. Additionally, because this performance task is based on the general requirements for graduation, it should not be seen as a modification to coursework and is therefore able to be made available to all students with disabilities.¹

¹ This performance task may not be appropriate for some students with disabilities based on their needs. It is an IEP team decision whether students with disabilities are working toward coursework requirements tied to this initiative and congruent with a standard diploma that meets federal definitions, an alternate pathway to a diploma, or a certificate of completion.

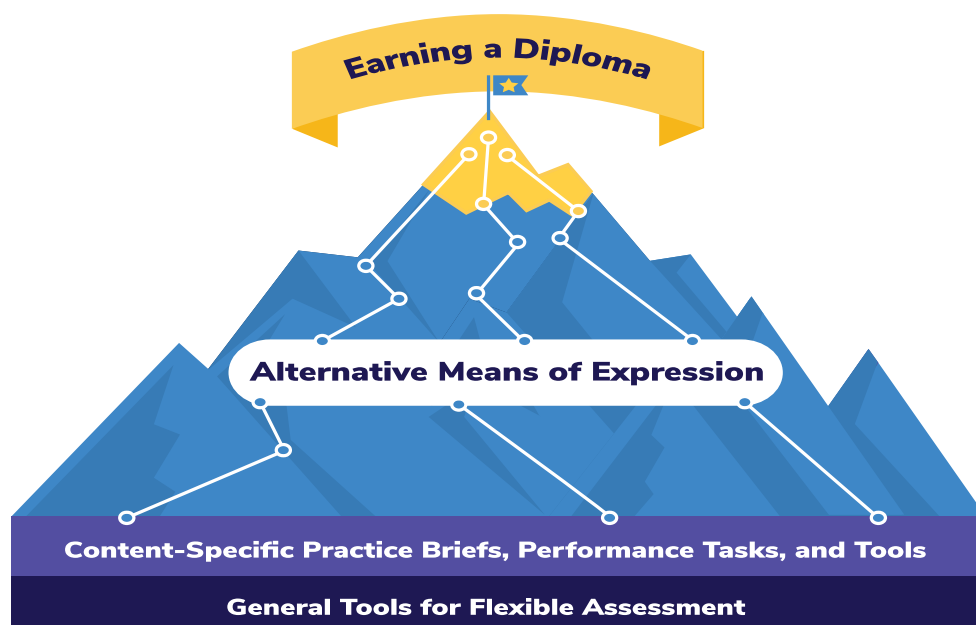




Multiple Routes to Proficiency

Figure 1 provides the conceptual framework for the *Inclusive Access to a Diploma: Reimagining Proficiency for Students with Disabilities* initiative. The graphic displays a mountain with the peak representing coursework requirements for earning a diploma, and the routes up the mountain representing different alternative means of expression made available to students for showing their understanding. The first bar at the foundation of the mountain represents the documents, materials, and resources—such as content-specific practice briefs, performance tasks, and assessment tools—serving as sample alternative means of expression. The second bar at the foundation of the mountain references the general tools, such as the Best Practice Guide, created through this initiative to support schools and districts in providing flexible assessment models.

Figure 1. Conceptual Framework for Inclusive Access to a Diploma Initiative





Overview of the Performance Task

This performance task evaluates students' understanding of key concepts within the Mathematics I Shapes and Structures, Building with Triangles, and Transformations and Congruence Big Ideas. It is divided into parts, with each targeting specific overlapping components of these three Big Ideas related to plane geometry. Each part offers accessible strategies and examples of how students can demonstrate proficiency with the concepts. Various tools, mediums, and connections are provided for teachers to customize the task to the unique needs, cultures, interests, and abilities of their students to promote an inclusive and relevant educational experience.

When preparing to administer this performance task, the teacher needs to distinguish between flexible and fixed elements to ensure students have multiple ways to demonstrate their knowledge without compromising the depth and rigor of the standards. As always, educators should also consult the student's Individualized Education Program (IEP) to ensure that all required accommodations and supplementary aids are provided during the assessment.

Additional information on providing alternative means of expression can be found in the Best Practice Guide and content-aligned Practice Briefs created as part of the *Inclusive Access to a Diploma: Reimagining Proficiency for Students with Disabilities* initiative.





Administering the Performance Task and Embedding Resources for Students

Each part of this task is broken into a series of items for administration. This section provides guidance to the educator on how to administer each part of the task and support the student in demonstrating their understanding of the Big Idea. As you plan to administer this performance task, it is suggested to review these recommendations as they offer associated key vocabulary, appropriate and inappropriate resources, and potential methods and means of expression.

Key Vocabulary Associated with the Standards

Understanding and correctly using the key vocabulary terms below is essential to demonstrating proficiency with the concepts that fall under these Big Ideas; thus, unless otherwise noted, this vocabulary cannot be taught during completion of the task. However, students may still use tools and resources embedded in the environment (such as word walls, notebooks, word banks, glossary, and so forth) for support with these words.

- coordinate points, square, parallel lines, perpendicular lines, distance formula, Pythagorean Theorem, transformation and rigid transformations, perimeter, slope, translation, reflection, line of reflection, rotation, center of rotation congruence, polygon, hypotenuse, angle of rotation, pre-image and image, corresponding points, and line segments

Strategies for Supporting Students

The following sections describe appropriate and inappropriate resources to provide students as they complete a task.





Appropriate Resources

Appropriate resources maintain the rigor of the standards while also accommodating any student difficulties such as confusion or anxiety while providing a resource the student could use to complete the task.

- reading the item to the student
- answering clarifying questions related to the key vocabulary, for example
 - Part 1 “coordinate points” mean writing a point as (x, y) , “vertex” is the corner point of a square, “perimeter” is the length all around a shape
 - Part 2 “polygon” is a shape with sides as line segments and vertices connecting these line segments
 - Part 3 “What are corresponding pre-image and image points?”
- helping the student to make sense of the item by asking questions such as, “What is this question asking you to figure out? What important information does the question give you? Are there any words you want to ask about or look up?”
- offering drawing tools (graph paper, pencil, colored pencils, highlighters, straight edge, compass, protractor, computer drawing technology, tracing paper, multiple copies of the same provided graphs, adaptive materials, assistive technology)
- helping the student to *access resources* by reminding them of the meaning of mathematical terms
- providing multiple copies of the graphed images (on large graph paper)
- providing manipulatives to design polygon $ABCDE$ in Part 2² and $\triangle ABC$ in Part 3³

² Using a cutout of a shape or tracing paper might support students in seeing congruence based on the same shape and size of the shapes, but that alone does not prove congruence in a mathematically rigorous way (that is, that one shape can be transformed into the other using a series of rigid transformations). Students will still need to demonstrate their understanding of how rigid transformations are connected to congruence.

³ Using a cutout of a shape or tracing paper might support students in seeing the transformation, but in order to show proficiency they must still demonstrate their





- printing images on larger sheets of graph paper
- allowing students to complete different parts or items over an extended period (versus completing an entire task or part in one sitting)

Inappropriate Resources

The list of inappropriate resources below identifies what assistance should be avoided, as these supports interfere with the student's independent completion of the task and may compromise its utility in terms of assessing the student's proficiency with the standards:

- explaining to students how to use resources such as a compass or protractor
- reteaching mathematics concepts (such as the Pythagorean Theorem or distance formula, attributes of rigid transformations)
- demonstrating how to solve a similar problem so the student can reproduce the teacher's strategy
- explaining to the student step-by-step how to complete the items
- breaking the problems down into a step-by-step process⁴

Potential Alternative Means of Expression

Potential methods and means of expression show the various ways students can demonstrate their knowledge of the standards being assessed in this part of the task.

The following options provide additional ways students might demonstrate their knowledge of the standards being assessed.

Students can complete this task by

understanding of how to specify each transformation and the properties of each transformation.

⁴ Part of what this task is assessing is the student's ability to read a contextual problem and formulate a mathematical strategy for solving it.



- using graph paper,⁵ pen or pencil, colored pencils or highlighters, straight edge, compass, and/or protractor
- using scientific or graphing calculators
- using graphing technology such as GeoGebra or Desmos
- annotating a paper or digital copy of a geometry image
- using verbal expression or text-to-speech software to describe the results (for example, in Part 3 describing why and how they would perform a transformation)
- dictating to a scribe⁶

⁵ The use of graph paper and pen or pencil is demonstrated in the sample responses.

⁶ In this situation, it is important for the scribe to be careful to record **only** what the student explicitly communicates, rather than making interpretations and “filling in the blanks” based on what they think the student meant.





PART 1. Using Coordinates to Prove Theorems and Solve Problems

Part 1 of this performance task includes

- associated standards that will be assessed
- student task requirements
- rubrics that assess each item
- sample student responses

Teachers should familiarize themselves with the related standards, review the student task, analyze each item's rubric, and view the sample student responses to sufficiently prepare for using the task to assess students' proficiency with the standards addressed in the task. Additionally, teachers must be careful to incorporate any IEP-defined supplementary aids and services specific to individual students with disabilities taking this performance task.

Task Alignment to Key Elements of Big Ideas and Standards

Clusters of content standards exist within the Big Ideas, allowing the Big Ideas to demonstrate the central concepts and key understandings of the course content. The indicator statements provide the teacher with the key concepts being evaluated in each Big Idea as well as the associated content standards centered within the Big Idea of this task and come from the *2023 Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (Math Framework)* and are aligned to California adopted mathematics state standards.

Shapes in Structure: Big Idea Indicator 2

Study the changes in coordinates and express the changes algebraically.



Building with Triangles: Big Idea Indicator 1

Investigate geometric figures, constructing figures in the plane, relating the distance formula to the Pythagorean Theorem, and noticing how areas and perimeters of polygons change as the coordinates change.

Transformations and Congruence: Big Idea Indicator 2

Investigate transformations in the plane. Use geometry software to study transformations, developing definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, and parallel lines.

Transformations and Congruence: Big Idea Indicator 3

Express translations algebraically.

Related Standards

- **Use coordinates to prove simple geometric theorems algebraically.** [Include distance formula; relate to Pythagorean Theorem]
 - *(Item 1a)* **G-GPE.4** Use coordinates to prove simple geometric theorems algebraically.
 - *(Item 1a)* **G-GPE.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (for example, find the equation of a line parallel or perpendicular to a given line that passes through a given point).
 - *(Item 1b)* **G-GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, for example, using the distance formula.



Part 1. Items

The focus of this task is for students to show their understanding of shapes using proof and algebraic knowledge.

Item 1. Directions [Student Document, p. 1]

Item 1 has two tasks.

Item 1 Tasks

The following coordinate points are three vertices of a square:

$(-5, 4)$

$(-7, -1)$

$(0, 2)$

Use this knowledge to complete the following items.

- (A) What are the coordinates (x, y) of the fourth vertex of this square? Without using a ruler or protractor to measure, prove that this shape is a square.
- (B) Find the perimeter of the square. Show all calculations.

A Rubric for Assessing a Response to Item 1a

G-GPE.4 Use coordinates to prove simple geometric theorems algebraically.

G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (for example, find the equation of a line parallel or perpendicular to a given line that passes through a given point).





Rubric for Part 1, Item 1a

Attempted	Approaching	Proficient
<p>The student attempts to graph the three given points and/or to find a fourth point.</p> <p>The justification is missing or so unclear that it is not possible to determine the student's level of understanding.</p>	<p>The student finds the fourth point of the square $(-2, -3)$.</p> <p>The student attempts to justify mathematically why the points form a square, but the justification is only partially correct or not fully thorough (for example, only considering slope but not connecting this to show how the lengths are congruent; or discussing how the sides are parallel but not showing they are congruent; or they created a graph that looks like a square but do not rigorously prove that the angles are right).</p>	<p>The student finds the fourth point of the square $(-2, -3)$.</p> <p>The student rigorously proved that the four points create a square by showing that</p> <p>all four side lengths are congruent, and</p> <p>all four angles are right angles.</p>

A Rubric for Assessing a Response to Item 1b

G-GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, for example, using the distance formula.

Rubric for Part 1, Item 1b

Attempted	Approaching	Proficient
<p>The student attempts to find the lengths but the answers are incomplete or incorrect.</p>	<p>The student response shows calculations for the distance of each length using either the distance formula or Pythagorean Theorem and then uses this to calculate the final perimeter of the square, with minor calculation errors. The student knows how to use the distance formula or Pythagorean Theorem.</p>	<p>The student response shows calculations for the distance of each length using either the distance formula or Pythagorean Theorem and then, uses this to calculate the final perimeter of the square.</p>



Part 1. Sample Student Responses

The content below provides a sample of proficient responses from a student. The text that leads with “Student Voice” is an example of how a student might respond to each item. This section should only serve as a model—different students will arrive at solutions in different ways.

Item 1 Tasks [Student Document, p. 1]

The following coordinate points are three vertices of a square:

$(-5, 4)$

$(-7, -1)$

$(0, 2)$

Use this knowledge to complete the following items.

- (A) What are the coordinates (x, y) of the fourth vertex of this square? Without using a ruler or protractor to measure, prove that this shape is a square.
- (B) Find the perimeter of the square. Show all calculations.

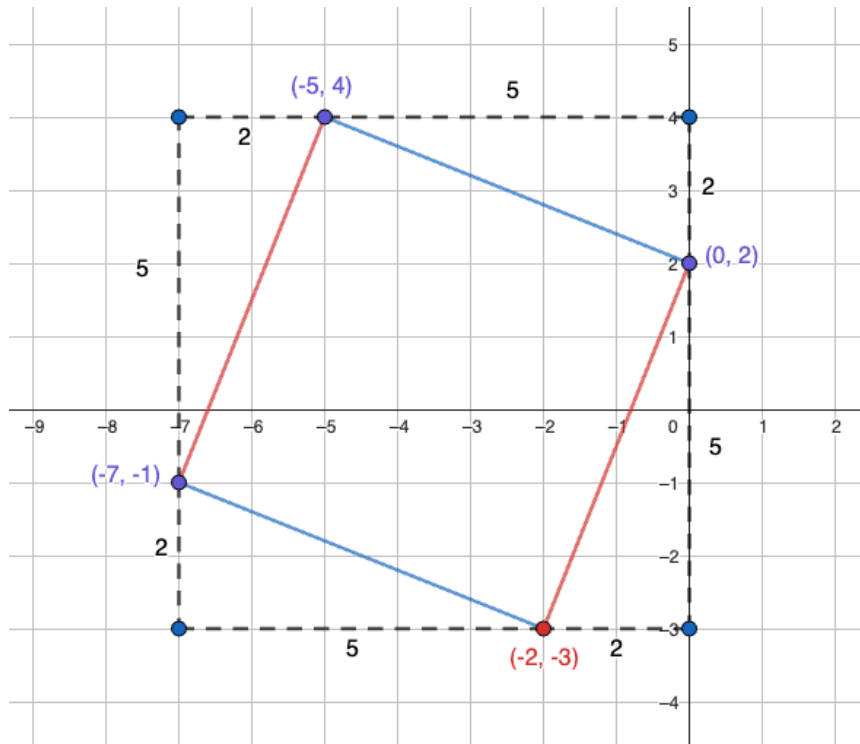
Correct Response for Part 1, Item 1a

Student Voice: The fourth point is $(-2, -3)$.

In this graph, the student graphed the 3 given points, $(-5, 4)$, $(-7, -1)$, and $(0, 2)$, creating a red line segment between $(-5, 4)$ and $(-7, -1)$ and a blue line segment between $(-5, 4)$ and $(0, 2)$. They then created slope triangles for each of these line segments and noted the lengths of each of the vertical and horizontal line segments of these triangles. The student then created a slope triangle from the point $(-7, -1)$ that mirrored the line segment’s slope triangle from $(-5, 4)$ and $(0, 2)$, going down 2 and 5 to the right, landing at the point $(-2, -3)$. Similarly, they also created a slope triangle from the point $(0, 2)$ that mirrored the line segment’s slope triangle from $(-5, 4)$ and $(-7, -1)$, going down 5 and 2 to the left, also landing at the point $(-2, -3)$, which is their fourth point of the square.



Figure 2. Sample Student-Generated Response



Student Voice: To prove that $(-2, -3)$ creates a square with the given points $(-5, 4)$, $(-7, -1)$, and $(0, 2)$, I wanted to show that all four sides are equal length and that all four angles are 90° .

All four sides are congruent:

I used the distance formula (or Pythagorean Theorem) to find the distance between the points $(-5, 4)$ and $(-7, -1)$:

$$\sqrt{(-5 - -7)^2 + (4 - -1)^2} = d$$

$$\sqrt{(2)^2 + (5)^2} = d$$

$$\sqrt{4 + 25} = d$$

$$\sqrt{29} = d$$

Similar calculations are shown for the distance of the other three lengths all equaling $\sqrt{29}$:

$$\text{distance between } (-7, -1) \text{ and } (-2, -3) = \sqrt{29}$$



distance between $(-2, -3)$ and $(0, 2) = \sqrt{29}$

All four angles are 90° :

I know that perpendicular lines, lines that intersect and give a 90° , have slopes that are the opposite reciprocal of each other, so if the slope of the sides between $(-7, -1)$ and

$(-5, 4)$ and the side between $(0, 2)$ and $(-2, -3)$ is $\frac{5}{2}$, then the other two sides must

have a slope of $-\frac{2}{5}$ to be opposite reciprocals and are then therefore right angles.

Looking at the graph, I used a slope triangle to confirm that the slopes of the sides $(-7, -1)$ and $(-2, -3)$ is $-\frac{2}{5}$ and so is the slope between $(-2, -3)$ and $(0, 2)$.

Other possible correct responses

The bullet points below describe other potential response types.

- Students might also use the distance formula instead of the Pythagorean Theorem. Students may use the distance formula and Pythagorean Theorem synonymously.
- Students might also use input-output tables to find the slope.
- They might also use their knowledge of transformations to find the point, such as
 - o translating the top side of the square down 5 and 2 to the right, to get the fourth point;
 - o since translating the points $(-5, -4)$ to $(-7, -1)$ is $(x-2, y-5)$, then the new point would be $(0-2, 2-5) = (-2, -3)$; or
 - o rotating the left side of the square clockwise 90° means that since the slope is $\frac{5}{2}$ the slope from $(-7, -1)$ needs to be $-\frac{2}{5}$, moving down two and 5 to the right to get $(-2, -3)$.





Correct Response for Part 1, Item 1b

Student Voice: Using the Pythagorean Theorem (or distance formula), I found the length of each side.

Use the slope triangle lengths of 2 and 5, to find the hypotenuse length:

$$(2)^2 + (5)^2 = c^2$$

$$4 + 25 = c^2$$

$$29 = c^2$$

$$c = \sqrt{29}$$

Since all four sides have slope triangles that are 2 and 5, the side lengths will each be $\sqrt{29}$ units.

So, the perimeter = $\sqrt{29} + \sqrt{29} + \sqrt{29} + \sqrt{29} = 4\sqrt{29}$ units (students might also calculate ≈ 21.54 units).





PART 2. Congruence, Rigid Motion, and Constructions

Part 2 of this performance task includes

- associated standards that will be assessed
- student task requirements
- rubrics that assess each item
- sample student responses

Teachers should familiarize themselves with the related standards, review the student task, analyze each item's rubric, and view the sample student responses to sufficiently prepare for using the task to assess students' proficiency with the standards addressed in the task. Additionally, teachers must be careful to incorporate any IEP-defined supplementary aids and services specific to individual students with disabilities taking this performance task.

Task Alignment to Key Elements of Big Ideas and Standards

Clusters of content standards exist within the Big Ideas, allowing the Big Ideas to demonstrate the central concepts and key understandings of the course content. The assessment indicators provide the teacher with the key concepts being evaluated in each Big Idea as well as the associated content standards centered within the Big Idea of this task and come from the 2023 *California Mathematics Framework* and are aligned to California-adopted mathematics state standards.

Shapes in Structure: Big Idea Assessment Indicator 1

Perform investigations that involve building triangles and quadrilaterals, considering how the rigidity of triangles and non-rigidity of quadrilaterals influence the design of structures and devices.





Transformations and Congruence: Big Idea Assessment Indicator 2

Explore congruence of triangles, including quadrilaterals built from triangles, through geometric constructions.

Related Standards

The following are standards that align with the assessment indicator items above:

- **Understand congruence in terms of rigid motions.** [Build on rigid motions as a familiar starting point for development of the concept of geometric proof.]
 - *(Items 2a, 2b)* **G-CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
 - *(Items 2a, 2b)* **G-CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Part 2. Items

The focus of this item is for students to use their understanding of rigid motions to determine or create congruence between shapes (**G-CO.6, G-CO.7**).

The diagram below shows three polygons you will use as you construct your responses.

Item 1. Directions [Student Document, p. 2]

Item 1 has two tasks. Use student document figure 1 to answer each task.

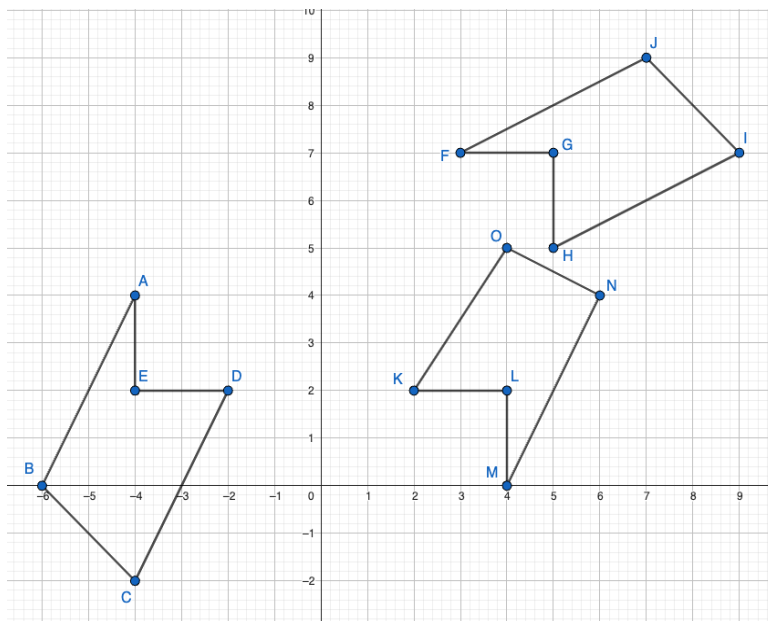




Item 1 Tasks

- (A) Which of the three polygons are congruent in figure 1? Show how you know using your understanding of congruence and rigid transformations. (Be sure to name the polygons in order of corresponding vertices.)
- (B) For the polygon that is not congruent to the other two in figure 1, explain how to modify only the not congruent polygon to make it congruent to the other two polygons.
- Change one or two of the vertices in the not congruent polygon
 - List the new vertex or vertices with the new coordinate points, (x, y)
 - State why this now makes all three polygons congruent.

Figure 3. Diagram of Three Polygons [Student Document, Figure 1, p. 3]



A Rubric for Assessing a Response to Item 1a

G-CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.



G-CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Rubric for Part 2, Item 1a

Attempted	Approaching	Proficient
The student attempts to explain whether the polygons are congruent or not, but they do not use rigid transformations to verify congruence.	<p>The student attempts to use transformations to describe why polygons $ABCDE$ and $FJIHG$ are congruent or why polygon $ABCDE$ uses a generally correct, mathematically rigorous strategy.</p> <p>The explanation may lack clarity, specificity, or <i>thoroughness</i> (for example they only show how some of the points are transformed but not all; transformations might be described via visual estimation instead of via mathematically rigorous descriptions and drawings).</p>	<p>The student response proves that polygons $ABCDE$ and $FJIHG$ are congruent.</p> <p>The student justifies this congruence because the same series of transformations (using any combinations of translation, reflection, or rotations for every corresponding point or all corresponding sides) can be used to transform all vertices in $ABCDE$ into the corresponding vertices in $FJIHG$.</p> <p>The student shows their transformations and demonstrates that the same series that works for all points and results in congruent parts.</p>



A Rubric for Assessing a Response to Item 1b

G-CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G-CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Rubric for Part 2, Item 1b

Attempted	Approaching	Proficient
The student attempts to say which polygons are not congruent, however, they do not use rigid transformations to verify why they are not congruent.	The student attempts to use transformations to describe that polygon $ABCDE$ is not congruent to polygon $MNOKL$ but they may not have a thorough explanation (for example they only show how some of the points are transformed but not all; or they try to transform through visual estimation instead of through mathematical descriptions of how they know if it is reflected correctly; they may not correct the out-of-place point using transformations, or the polygons are not named in the order of corresponding points).	<p>The student identifies the correct change to polygon $MNOKL$ (moving point O from $(4, 5)$ to $(4, 6)$).</p> <p>The student gives a mathematically rigorous explanation for why this change will result in a polygon that is congruent to $ABCDE$ and/or $FJIHG$.</p>



Part 2. Sample Student Responses

The content below provides a sample of proficient responses from a student. The text that leads with “Student Voice” is an example of how a student might respond to each item. This section should only serve as a model—different students will arrive at solutions in different ways.

Item 1 Tasks [Student Document, p. 2]

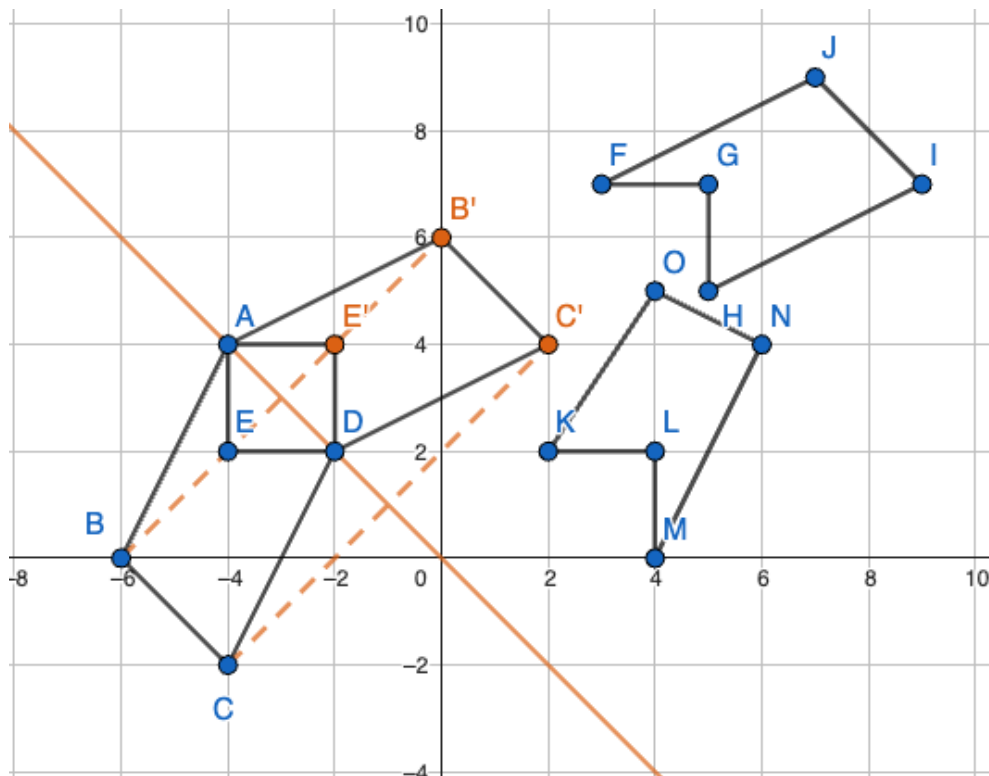
- (A) Which of the three polygons are congruent in figure 1? Show how you know using your understanding of congruence and rigid transformations. (Be sure to name the polygons in order of corresponding vertices.)
- (B) For the polygon that is not congruent to the other two in figure 1, explain how to modify only the not congruent polygon to make it congruent to the other two polygons.
- Change one or two of the vertices in the not congruent polygon
 - List the new vertex or vertices with the new coordinate points, (x, y)
 - State why this now makes all three polygons congruent.



Correct Response for Part 2, Item 1a

Student Voice: Polygon $ABCDE$ is congruent to polygon $FJIHG$. First, I can reflect polygon $ABCDE$ across the line $y = -x$ (or across the line \overleftrightarrow{AD}). Because it is a reflection, the new polygon $A'B'C'D'E' \cong$ polygon $ABCDE$, where $A'(-4, 4)$, $B'(0, 6)$, $C'(2, 4)$, $D'(-2, 2)$, $E'(-2, 4)$. Figure 4 provides an annotated example of these concepts.

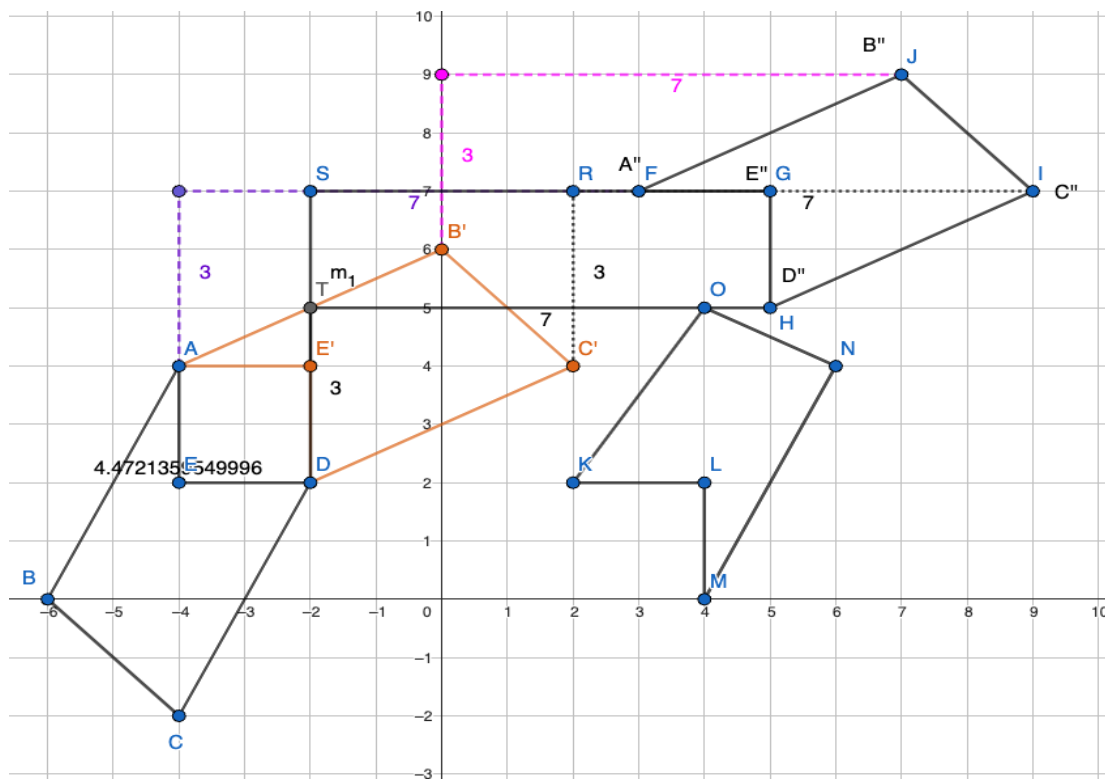
Figure 4. Sample Student-Generated Diagram





Student Voice: Next, I can translate polygon $A'B'C'D'$ 3 units up and 7 units to the right. F becomes A'' , J becomes B'' , I becomes C'' , H becomes D'' , and G becomes E'' . Since I can create polygon $FJIHG$ from $ABCDE$ with a reflection and a translation, the new polygon $FJIHG \cong$ polygon $ABCDE$.

Figure 5. Sample Student-Generated Diagram

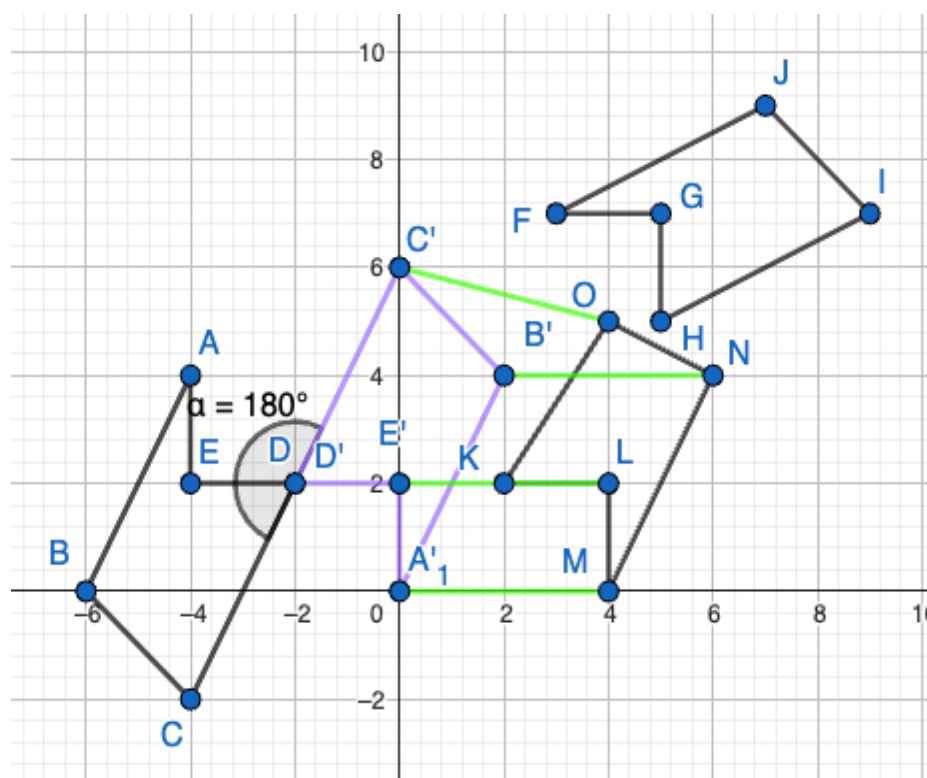




Student Voice: Polygons $ABCD$ and $FJIHG$ are not congruent to polygon $MNOKL$. First, I rotated the polygon $ABCD$ 180° clockwise around point D to get my polygon. This new polygon $A'B'C'D'$ (at $A'(0, 0)$, $B'(2, 4)$, $C'(0, 6)$, $D'(-2, 2)$, $E'(0, 2)$) is congruent to polygon $ABCD$ because rotations hold congruence.

Next, I made line segments to connect the corresponding vertices of polygon $A'B'C'D'$ to polygon $MNOKL$, which, if it was a rigid translation, then all these new line segments should be parallel and the exact same length. This worked for all points A' , B' , and D' which translated 4 units to the right, but point C' went at a diagonal, so polygon $MNOKL$ is not congruent to polygon $ABCD$ because point O is out of place.

Figure 6. Sample Student-Generated Diagram

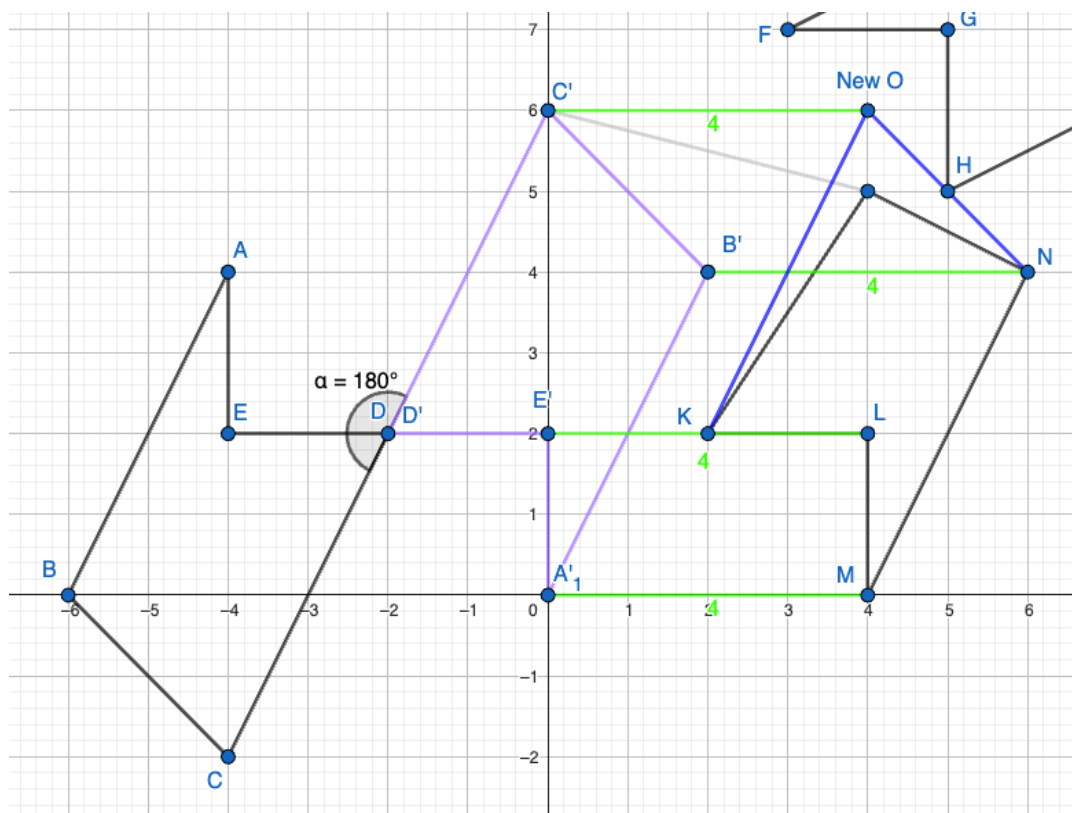




Correct Response for Part 2, Item 1b

Student Voice: To make polygon $ABCD \cong$ polygon $MNOKL$, I can continue my translation that worked for the other points and move C' four units to the right or $(x + 4, y)$ to get a new point O . C' is $(0, 6)$, so the new O should be at $(0 + 4, 6)$ or $(4, 6)$.

Figure 7. Sample Student-Generated Diagram





PART 3. Transformations in the Plane

Part 3 of this performance task includes

- associated standards that will be assessed
- student task requirements
- rubrics that assess each item
- sample student responses

Teachers should familiarize themselves with the related standards, review the student task, analyze each item's rubric, and view the sample student responses to sufficiently prepare for using the task to assess students' proficiency with the standards addressed in the task. Additionally, teachers must be careful to incorporate any IEP-defined supplementary aids and services specific to individual students with disabilities taking this performance task.

Task Alignment to Key Elements of Big Ideas and Standards

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Building with Triangles: Big Idea Assessment Indicator 2

Build with triangles and quadrilaterals, noticing positions and movement, and creating equations that model the changing edges using technology.





Transformations and Congruence: Big Idea Assessment Indicator 2

Investigate transformations in the plane. Use geometry software to study transformations, developing definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, and parallel lines.

Related Standards

- **Experiment with transformations in the plane.**
 - *(Items 1a, 2a, 3a)* **G-CO.2** Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (for example, translation versus horizontal stretch).
 - *(Items 2a, 3a)* **G-CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
 - *(Items 1b, 2b, 3b)* **G-CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Part 3. Items

For the various tasks in Part 3, a triangle in the upper left quadrant is labeled $\triangle ABC$. This triangle is then transformed by either a translation, a reflection, and/or a rotation, to produce three images in the other quadrants. Read each item and complete the tasks.

Item 1. Directions [Student Document, p. 4]

Item 1 has two tasks. Use student document figure 2 to answer each task.



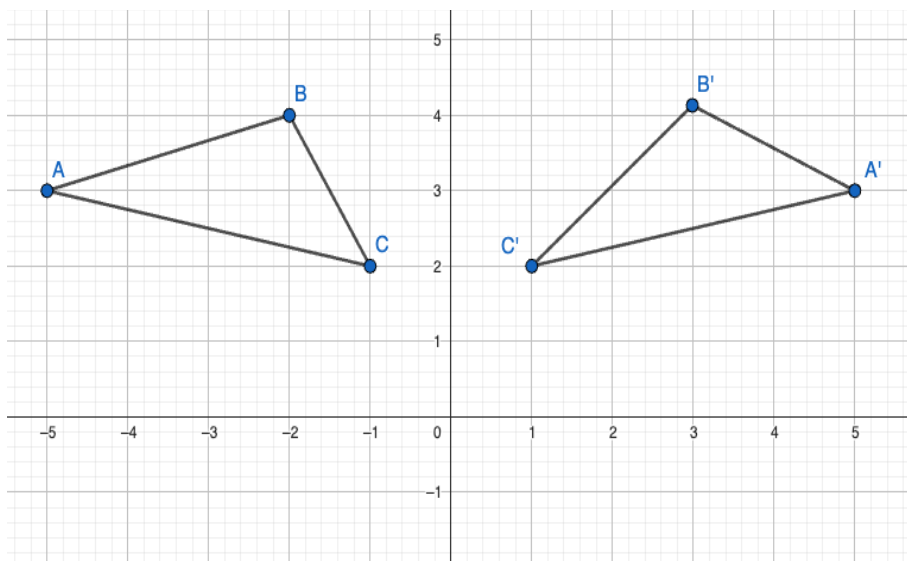


Item 1 Tasks

- (A) Describe the transformation of $\triangle ABC$ that results in $\triangle A'B'C'$ in figure 2. Be specific. (Include when appropriate: distance, direction, line of reflection, center of rotation, or angle of rotation.)
- (B) For the transformation from $\triangle ABC$ to $\triangle A'B'C'$ in figure 2, explain the properties of this transformation, using line segments between the pre-image and image points and other reference parts on the coordinate plane.

Figure 8. Triangle Images for Part 3, Item 1

[Student Document, Figure 2, p. 5]





A Rubric for Assessing Item 1a

G-CO.2 Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (for example, translation versus horizontal stretch).

Rubric for Part 3, Item 1a

Attempted	Approaching	Proficient
The student attempts to name a transformation.	The student correctly identifies the transformation as a reflection but does not specify the equation of the line of reflection (for example, “You flip it over a line so it is a mirror image”).	The student correctly states that $\triangle A'B'C'$ is a reflection of $\triangle ABC$ across the line of symmetry that is the y -axis (or the line $x = 0$).

A Rubric for Assessing Item 1b

G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Rubric for Part 3, Item 1b

Attempted	Approaching	Proficient
The student attempts to connect the image to pre-image points but it is not clear from the explanation that the student understands the properties that must be true for all reflections .	The student identifies some properties of a reflection , though the explanation may lack specificity and thoroughness (for example, they may connect corresponding pre-image to image points and possibly draw congruent marks on the segments but do not show that the measurement between the pre-image and image points and line of reflection are equidistant) rather than a mathematically rigorous way (for example, “the line of reflection is a perpendicular bisector for each line segment connecting the corresponding points from the pre-image and image”).	<p>The student correctly states that in a reflection, the line of reflection is a perpendicular bisector for each line segment connecting the corresponding points from the pre-image and image points.</p> <p>The student gives a rigorous mathematical explanation for how they know this is the case in this problem.</p>



Item 2. Directions [Student Document, p. 6]

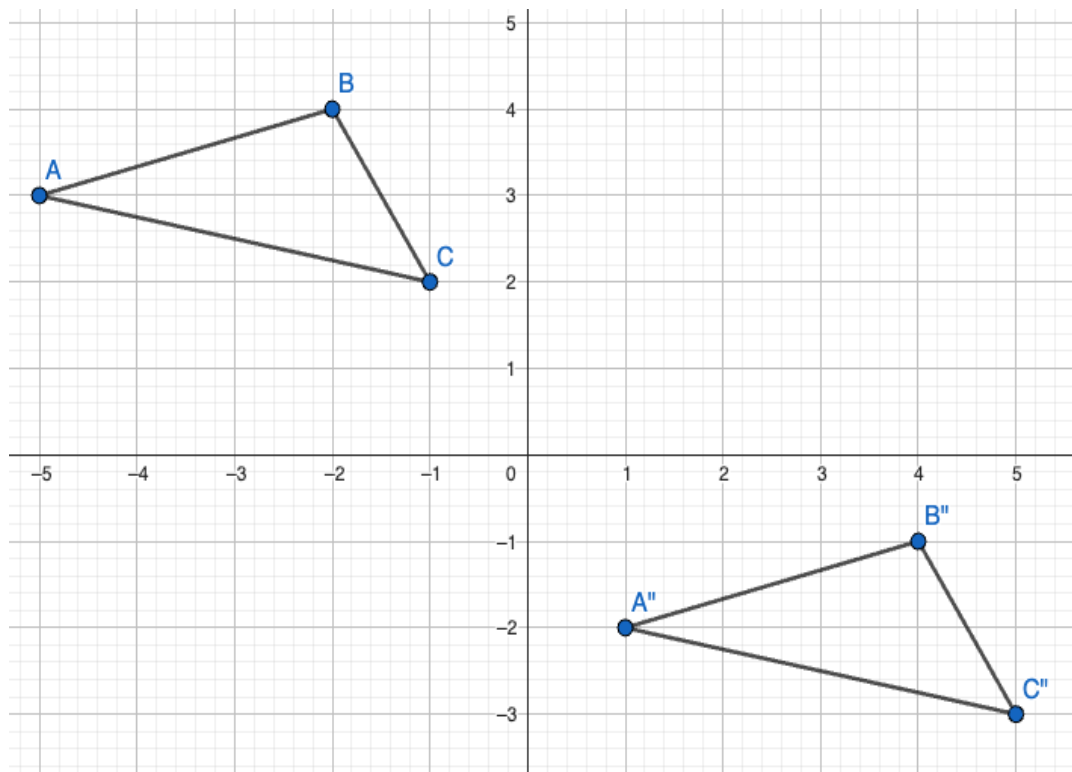
Item 2 has two tasks. Use student document figure 3 to answer each task.

Item 2 Tasks

- (A) Describe the transformation of $\triangle ABC$ that results in $\triangle A''B''C''$ in figure 3. Be specific (include when appropriate: distance, direction, line of reflection, center of rotation, or angle of rotation).
- (B) For the transformation from $\triangle ABC$ to $\triangle A''B''C''$ in figure 3, explain the properties of this transformation, using line segments between the pre-image and image points and other reference parts on the coordinate plane.

Figure 9. Triangle Images for Part 3, Item 2

[Student Document, Figure 3, p. 7]





A Rubric for Assessing Item 2a

G-CO.2 Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (for example, translation versus horizontal stretch).

G-CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Rubric for Part 3, Item 2a

Attempted	Approaching	Proficient
The student attempts to name a translation .	The student correctly identifies the transformation as a translation but does not specify the number of units or the directions of the translation.	The student correctly states that $\triangle A''B''C''$ is a translation of $\triangle ABC$ and describes this as 6 units to the right and 5 units down.

A Rubric for Assessing Item 2b

G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Rubric for Part 3, Item 2b

Attempted	Approaching	Proficient
The student attempts to connect the image to pre-image points but does not make a connection to the properties that must be true for all translations.	The student identifies some properties of a translation , though the explanation may lack specificity and thoroughness (for example, they might say only one of the attributes of this property, “the line segments connecting the corresponding points from the pre-image and image are congruent,” without mentioning they are also parallel, or say both attributes of the property without rationale for how they know this is true, “the line segments are congruent” without a mathematically rigorous justification).	<p>The student correctly states that in a translation, the line segments connecting the corresponding points from the pre-image and image will give congruent and parallel line segments.</p> <p>The student gives a rigorous mathematical explanation for why this is the case in this problem.</p>



Item 3. Directions

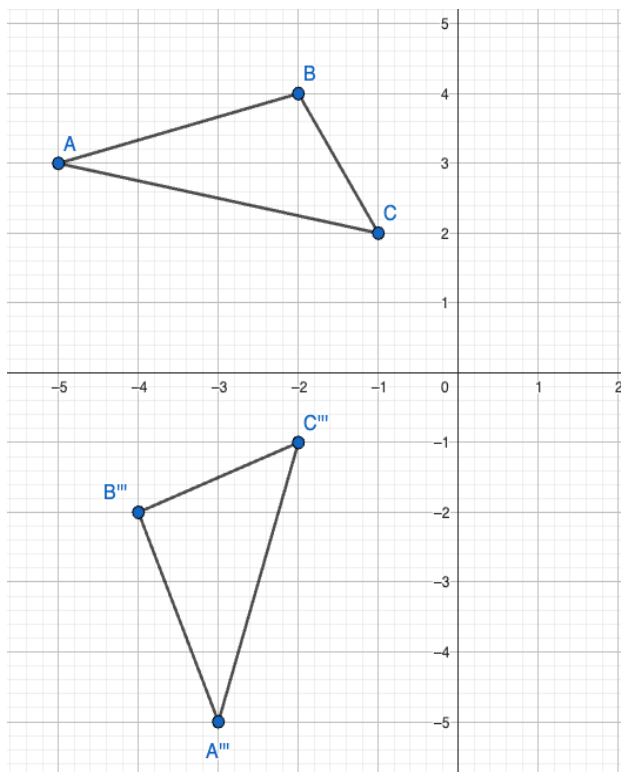
Item 3 has two tasks. Use student document figure 4 to answer each task.

Item 3 Tasks [Student Document, p. 8]

- (A) Describe the transformation of $\triangle ABC$ that results in $\triangle A'' B'' C''$ in figure 4. Be specific (include when appropriate: distance, direction, line of reflection, center of rotation, or angle of rotation).
- (B) For the transformation from $\triangle ABC$ to $\triangle A'' B'' C''$, explain the properties of the transformation in figure 4, using line segments between the pre-image and image points and other reference parts on the coordinate plane.

Figure 10. Triangle Images for Part 3, Item 3

[Student Document, Figure 4, p. 9]





A Rubric for Assessing Item 3a

G-CO.2 Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (for example, translation versus horizontal stretch).

G-CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Rubric for Part 3, Item 3a

Attempted	Approaching	Proficient
The student attempts to name a transformation but is incorrect.	The student correctly identifies this diagram as a rotation but does not specify the angle, direction, or center point.	The student correctly stated that $\triangle A''B''C''$ is a rotation of $\triangle ABC$ 90° counterclockwise around a center point of (0, 0).

A Rubric for Assessing Item 3b

G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Rubric for Part 3, Item 3b

Attempted	Approaching	Proficient
The student attempts to connect the image to pre-image points but does not make a connection to the properties that must be true for all rotations.	The student identifies some properties of a rotation , though the explanation may lack specificity and thoroughness (for example, they may not completely show that the line segments connecting the corresponding points from the pre-image and image each have a perpendicular bisector that intersects the center of rotation).	The student correctly states that in a rotation , the perpendicular bisectors of each of the line segments connecting the corresponding points from the pre-image and image will all intersect at the point of rotation (the origin in this case) AND is able to show that this is true for this problem.



Part 3. Sample Student Responses

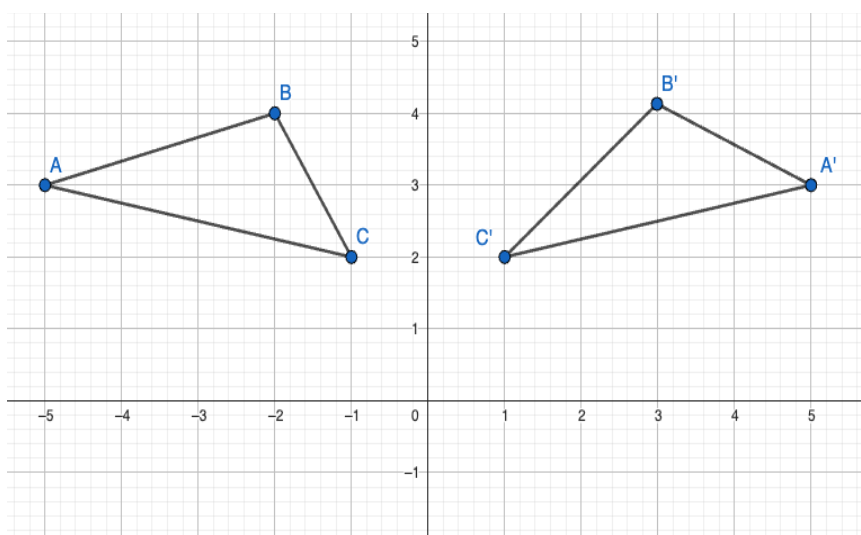
The content below provides a sample of proficient responses from a student. The text that leads with “Student Voice” is an example of how a student might respond to each item. This section should only serve as a model—different students will arrive at solutions in different ways.

Item 1 Tasks [Student Document, p. 4]

- (A) Describe the transformation of $\triangle ABC$ that results in $\triangle A'B'C'$ in figure 2. Be specific. (Include when appropriate: distance, direction, line of reflection, center of rotation, or angle of rotation.)
- (B) For the transformation from $\triangle ABC$ to $\triangle A'B'C'$ in figure 2, explain the properties of this transformation, using line segments between the pre-image and image points and other reference parts on the coordinate plane.

Figure 11. Triangle Images for Part 3, Item 1

[Student Document, Figure 2, p. 5]

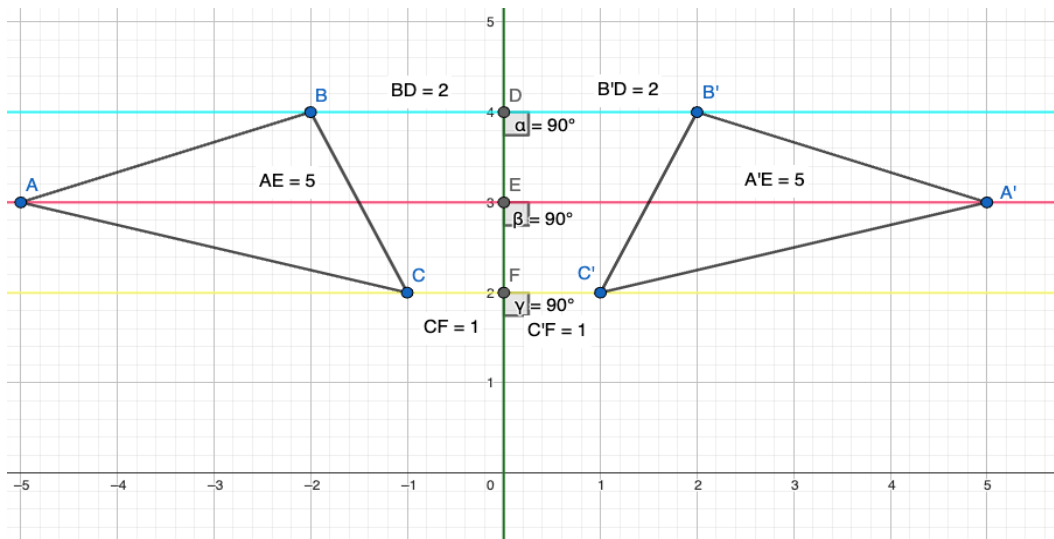




Correct Response for Part 3, Item 1a

Student Voice: The transformation from $\triangle ABC$ to $\triangle A'B'C'$ is a reflection of $\triangle ABC$ across the line of reflection that is the y -axis to get $\triangle A'B'C'$. See figure 12 for an example.

Figure 12. Student Annotated Diagram



Correct Response for Part 3, Item 1b

Student Voice: I know that for a reflection you need to specify a line of reflection. To be a reflection, the line of reflection will be the perpendicular bisector of the line segments created by connecting the corresponding pre-image to image points.

In my picture, figure 12, I connected A to A' , B to B' , and C to C' . Each of these lines crosses the y -axis or line of symmetry at 90° .

I also measured the distance between each pre-image/image point to the line of symmetry. You can see that these distances are congruent to the distance from the reflected point to the line of reflection. So, in my measurements, $BD = B'D' = 2$ units, $AE = A'E' = 5$ units, $CF = C'F' = 1$ unit.

Therefore, the y -axis is the perpendicular bisector of the segments connecting the corresponding pre-image to image points.



Other possible responses

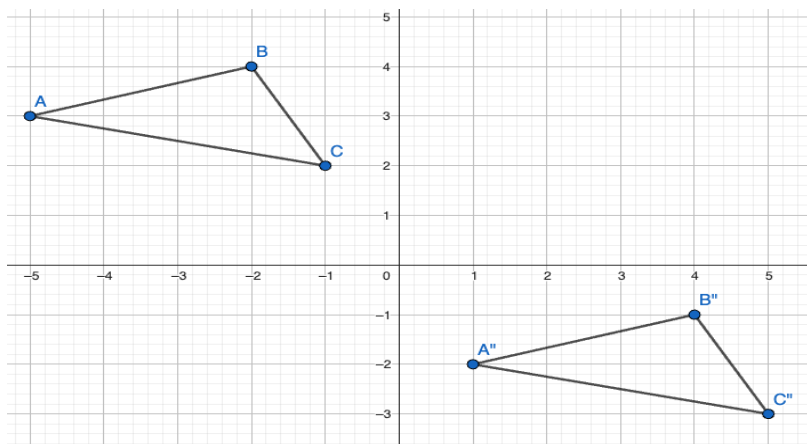
- Students might also say that the line segments created between corresponding points in the pre-image and image are all parallel. This will be true also, but this is also a default of each being perpendicular to the line of symmetry. Thus, if students have this additional statement that is wonderful noticing and they also need the perpendicular relationship and equidistance to the line of symmetry.
- Students might also say this is a reflection because for each point (x, y) the reflection point is $(x, -y)$. While this is true and enough to state it is a reflection, this does not satisfy explaining the properties of reflections.

Item 2 Tasks [Student Document, p. 6]

- (A) Describe the transformation of $\triangle ABC$ that results in $\triangle A''B''C''$ in figure 3. Be specific (include when appropriate: distance, direction, line of reflection, center of rotation, or angle of rotation).
- (B) For the transformation from $\triangle ABC$ to $\triangle A''B''C''$ in figure 3, explain the properties of this transformation, using line segments between the pre-image and image points and other reference parts on the coordinate plane.

Figure 13. Triangle Images for Part 3, Item 2

[Student Document, Figure 3, p. 7]

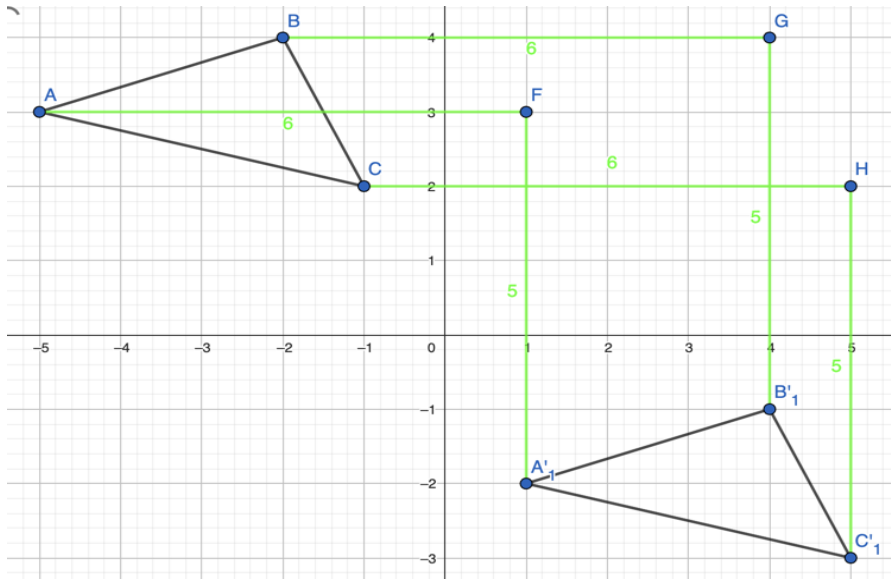




Correct Response for Part 3, Item 2a

Student Voice: $\triangle ABC$ to $\triangle A''B''C''$ is a translation 6 units to the right and 5 units down, which you can see in figure 14.

Figure 14. Student Annotated Diagram

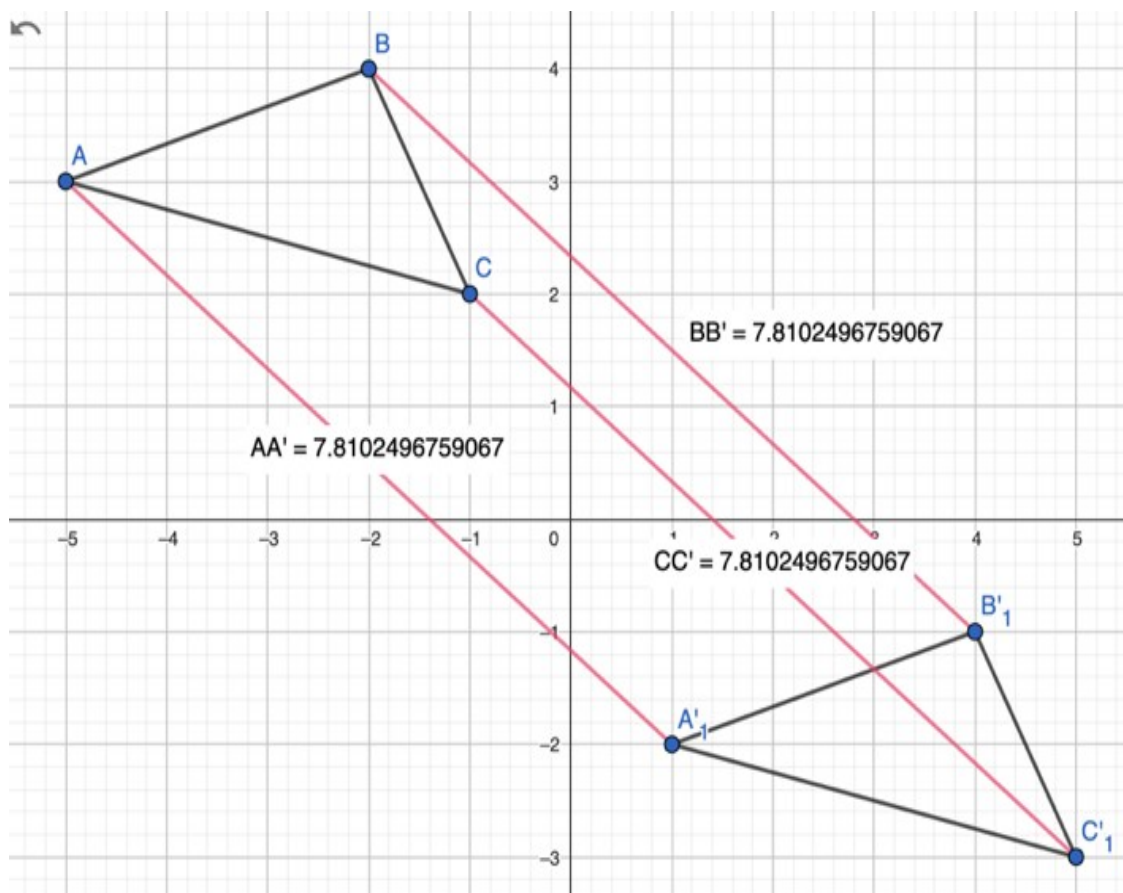




Correct Response for Part 3, Item 2b

Student Voice: I know this is a translation because when I connect the corresponding pre-image and image points, you see parallel line segments that are also congruent. My second picture in figure 15 below shows the parallel lines that each have a length of about 7.81 units (students can use dynamic geometry tools or rulers to confirm this, as well as the Pythagorean Theorem or distance formula).

Figure 15. Student Annotated Diagram



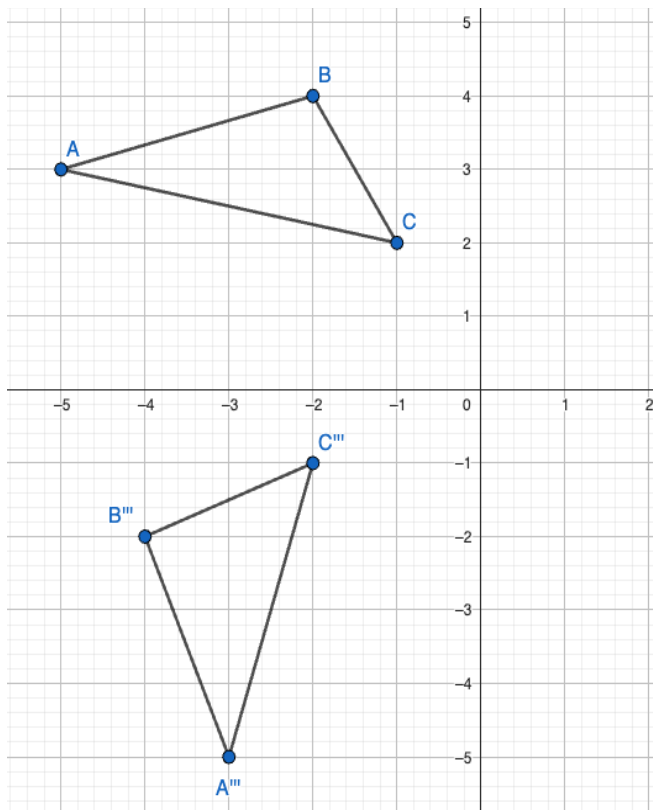


Item 3 Tasks [Student Document, p. 8]

- (A) Describe the transformation of $\triangle ABC$ that results in $\triangle A'' B'' C''$ in figure 4. Be specific (include when appropriate: distance, direction, line of reflection, center of rotation, or angle of rotation).
- (B) For the transformation from $\triangle ABC$ to $\triangle A'' B'' C''$, explain the properties of the transformation in figure 4, using line segments between the pre-image and image points and other reference parts on the coordinate plane.

Figure 16. Triangle Images for Part 3, Item 3

[Student Document, Figure 4, p. 9]

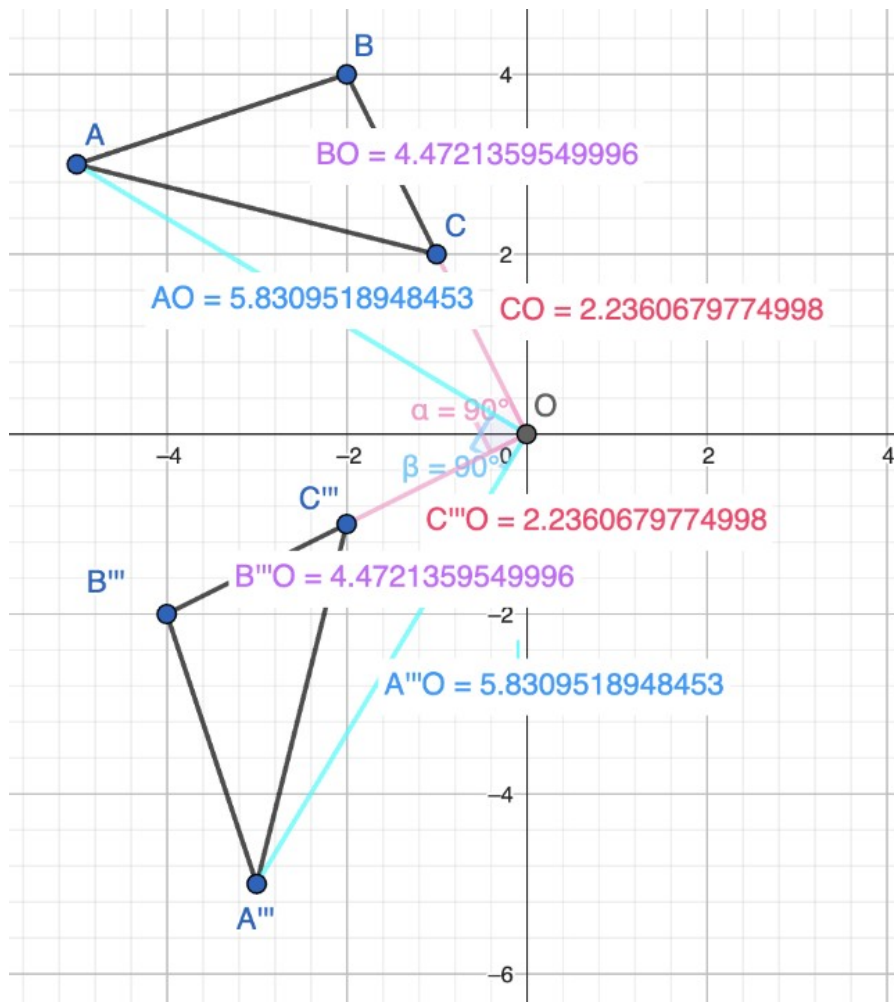




Correct Response for Part 3, Item 3a

Student Voice: This transformation of $\triangle ABC$ that results in $\triangle A'''B'''C'''$ is a rotation of 90° counterclockwise around a center point of $(0, 0)$ or the origin that you can see in figure 17.

Figure 17. Student Annotated Diagram





Correct Response for Part 3, Item 3b

Student Voice: When I created the line segments that connect each corresponding pre-image to image points, the perpendicular bisector for each of these will go through the center point of (0, 0) or the origin, which you can see in figure 18.

Figure 18. Student Annotated Diagram

