Algebra I – Functions Supertask

Model with Functions, Features of Functions, and Function Investigations

Option #1 Performance Task | Student Document

Authors: Initiative Team

April 30, 2024

Funded by the California Department of Education, Special Education Division

Directions

Please review the task below and answer the various questions within the task to the best of your ability. If needed, you may have an adult or peer read the task out loud to aid your understanding. You may use tools and embedded resources such as word walls, notebooks, word banks, and a glossary to support your understanding. These tools must be used independently without the support of a peer or teacher. Additionally, feel free to use the following resources in answering each item:

Once you have identified the function, you can evaluate the function in the following ways:

- mentally[1]
- handwritten using paper and pencil
- typed
- generated using speech-to-text software
- dictated to a scribe[2]
- explained verbally
- generated using an annotated graph or table to support or help clarify your explanation

[1] It is **not** appropriate for a student to use software to identify the function based on the output values; this item is in part intended to assess whether students can use the mathematics of linearity to determine the function themselves using a series of given input and output values.

[2] In this situation, it is important for the scribe to be careful to record **only** what the student explicitly communicates, rather than making interpretations and "filling in the blanks" based on what they think the student meant.

When creating graphs you can use the following tools:

- by hand, using paper and pencil
- using a graphing calculator
- using online graphing software such as Desmos or GeoGebra

Word Bank. Here are some words that might help you in explaining your thinking for the questions in this task. (You will probably not use every word for every question, so pick and choose the words that make sense for each question.)

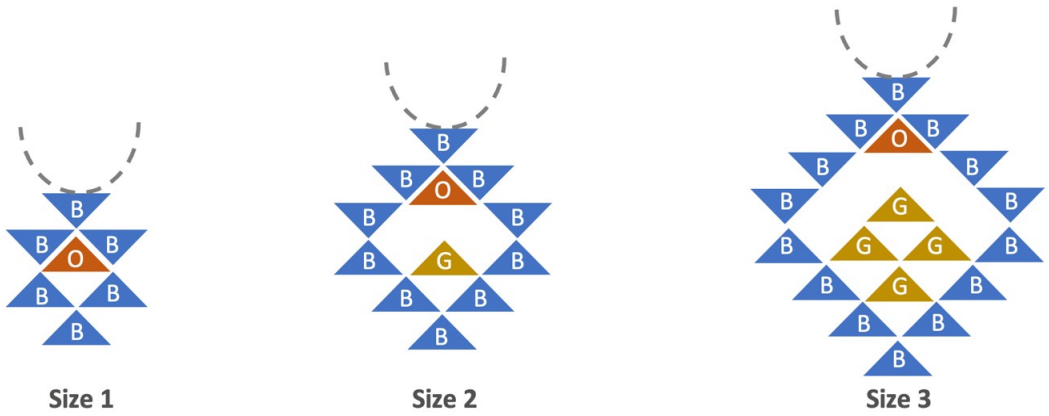
function	model	graph	input	output
variable	term	dependent variable	independent variable	point
coordinates	value	evaluate	substitute	linear
linear function	*x*-axis	*y*-axis	*y*-intercept	slope
rate of change	unit rate	common difference	coefficient	trendline
residuals	line of best fit	least squares method	correlation	correlation coefficient
exponential function	exponent	base	predict/ prediction	predicted value
interval	average rate of change	represent/ representation	causation	estimate
increase/ increasing	decrease/ decreasing	vertex	*x*-intercept	minimum
maximum	domain	parabola	factor	solve
zero product property (ZPP)				

PART 1. Ajana's Necklaces

Using Functions to Construct Models and Solve Problems

Ajana is starting a jewelry business. One of her necklace designs can come in many different sizes. Below are images of size 1, size 2, and size 3.

Figure 1. Ajana's Necklace Designs



Item 1. Directions

Item 1 has three tasks.

Let n represent the necklace size and let $b(n)$ represent the number of blue tiles that Ajana needs to make a necklace.

Item 1 Tasks

(A) Find $b(4)$ and $b(5)$; then give a recursive description of the function $b(n)$.

(B) Graph the function and label the graph and axes, find an explicit (closed) function $b(n)$ that models the number of blue tiles needed to make a necklace of size n, and explain how you found the function.

(C) Evaluate $b(100)$. Show or explain how you found it.

Item 2. Directions

Item 2 has two tasks.

Ajana wants to find a function she can use to figure out which size necklace she should make if she wants it to have a certain number of blue tiles. That is, she wants to find a function where the input x is "number of blue tiles" and the output $b^{-1}(x)$ is "necklace size."

Item 2 Tasks

(A) Use the function you found in Item 1 above to help her find the function she needs.

(B) Use the new function you found to determine what size necklace Ajana should make if she wants it to have 86 blue tiles, and what size she should make if she wants 52 blue tiles. Show and explain your reasoning.

Item 3. Directions

Item 3 has three tasks.

Let $g(n)$ represent the number of gold tiles that Ajana needs to make a necklace of size n.

Item 3 Tasks

(A) Find $g(4)$ and $g(5)$. Then give a recursive description of the function $g(n)$.

(B) Graph the function and label the graph and axes. Find an explicit (closed) function $g(n)$ that models the number of gold tiles needed to make a necklace of size n and explain how you found the function.

(C) Evaluate $g(100)$. Show or explain how you found it.

(D)

Item 4. Directions

Item 4 has two tasks.

Let $t(n)$ represent the total number of tiles that Ajana uses to make a necklace of size n.

Item 4 Tasks

(A) Graph the function and label the graph and axes. Find an explicit (closed) function $t(n)$ that models the number of blue tiles needed to make a necklace of size n and explain how you found the function.

(B) Evaluate $t(100)$. Show or explain how you found it.

PART 2. Jewelry Orders

Analyzing, Interpreting, and Constructing Functions

Item 1. Directions

Item 1 has four tasks.

Ajana wants to participate in more community markets so that she can sell more jewelry and connect with more customers. But participating in markets is a lot of time and work, and she is not sure whether it is worth it to attend more markets.

Ajana decides to look at some data from the past few years comparing the number of community markets she participates in each month with the number of orders she receives to see if the extra time and work is worth it.

To better understand the impact of participating in more community markets, she looks at some data from the past few years comparing the number of community markets she participates in each month with the number of orders she receives. This data is shown in table 1 below.

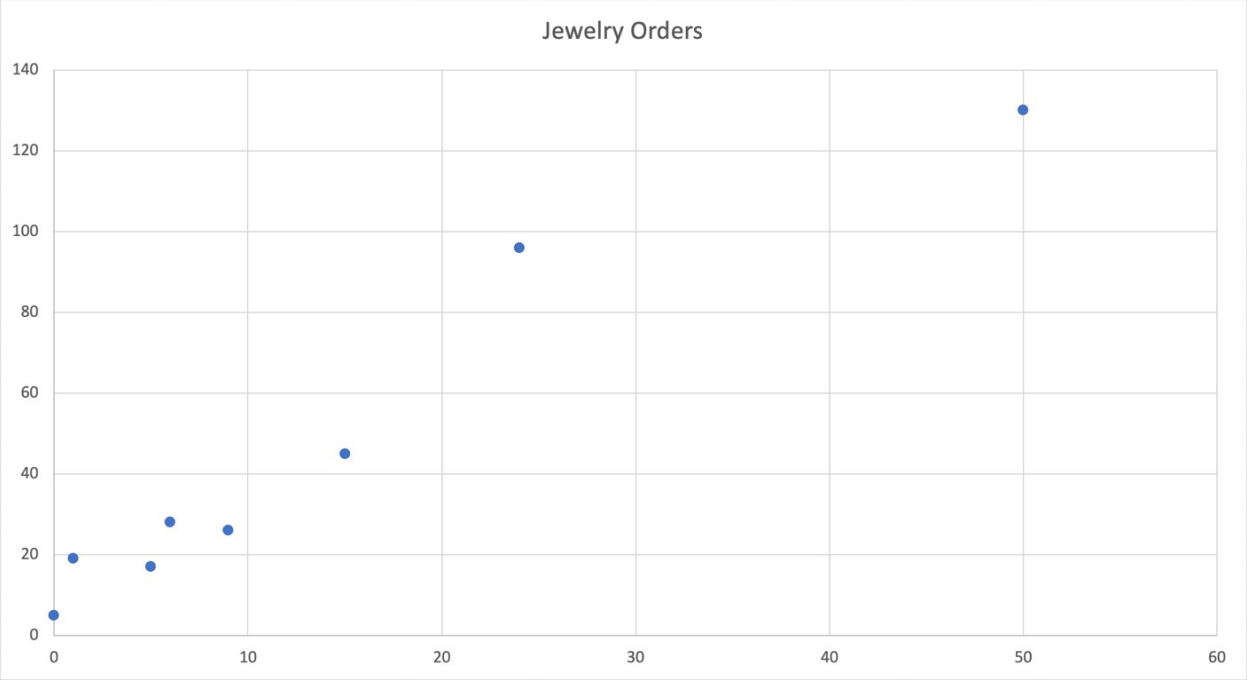
Then, Ajana uses the data to create a scatter plot (figure 2 below).

Table 1. Data on Jewelry Orders

Markets attended in one month	Jewelry orders
0	5
1	19
2	17
2	28
3	26
4	45
6	55

Markets attended in one month	Jewelry orders
7	96
15	130

Figure 2. Graph of Jewelry Orders



Item 1 Tasks

(A) Ajana thinks the data that looks like a linear function might be the best model for the relationship between the two variables.

- Let x represent the number of markets Ajana attends in a month and let $f(x)$ represent the number of jewelry orders she can expect to receive if she attends x markets per month. Then, use technology to find a function that you think models the relationship shown in the data.
- Explain and show using trendlines and residuals whether you think the relationship is linear or not.

(B) What does the linear model suggest about the relationship between the number of community markets Ajana attends and the number of jewelry orders? Would you consider the relationship between the number of markets she attends and jewelry orders to be strong, moderate, or weak? Explain your reasoning.

(C) Compute the correlation coefficient. Does the value of the correlation coefficient support your response to (Item 1b) above or not? Explain why or why not.

(D) Based on the linear model you found in (Item 1b), what is the estimated number of additional orders that Ajana can expect if she attends three additional markets in a given month? What is the y-intercept of the least squares line, and how would you interpret the meaning of this number in the context of the problem?

Item 2. Directions

Item 2 has three tasks.

After a few more months, Ajana added some new data to her table, shown in table 2 below.

Item 2 Tasks

(A) Add the following months of data to your scatter plot.

- If these new data points are included, would you still consider using a line to model the relationship? If not, what other type of model would you consider? Explain your reasoning.

Table 2. Markets and Orders Data

Markets	Orders
5	18
8	224
9	146

(B) Ajana decides to use the function $g(x)=0.02\cdot 3^x$ to model the relationship between the two variables. Graph the function $g(x)$. According to this model, how many orders could Ajana expect if she attends 5 markets? 10 markets? 12 markets?

(C) According to this model, how could Ajana figure out how many more orders can expect if she attends three additional markets in a given month? What is the y-intercept of the function, and how would you interpret the meaning of this number in the context of the problem?

Item 3. Directions

Item 3 has four tasks.

For each of the four situations below, identify the graph sketch and equation that match the situation and explain in words how you know. For the remaining situation, create the missing graph sketch and a possible table of values.

Item 3 Tasks

(A) At the market, Ajana's sales started slowly, then sped up. Around lunch time, sales slowed down again.

- Which table could match this situation—Table 3, 4, or 5? (see p. 9)
- Which graph sketch below could match this situation—Graph Sketch I, II, or III? (see figure 3, p. 10)

(B) For every small necklace Ajana sells, she makes the same amount of money.

- Which table could match this situation—Table 3, 4, or 5? (see p. 9)
- Which graph sketch below could match this situation—Graph Sketch I, II, or III? (see figure 3, p. 10)

(C) Ajana spent the day before the market making necklaces to sell. She started off working quickly, but as the day went on and she felt more and more tired, she made fewer and fewer necklaces.

- Which table could match this situation—Table 3, 4, or 5? (see p. 9)
- Which graph sketch below could match this situation—Graph Sketch I, II, or III? (see figure 3, p. 10)

(D) Over the course of the day, more and more people came to the market, but around lunch time, people slowly began to leave; as the afternoon went on, people started leaving more and more quickly until they were all gone.

- Which table could match this situation—Table 3, 4, or 5? (below)
- Which graph sketch below could match this situation—Graph Sketch I, II, or III? (see figure 3, p. 10)

Table 3. Function Table f

x	0	1	2	3	4	5	6	7	8
$f(x)$	0	49	84	105	112	105	84	49	0

Table 4. Function Table g

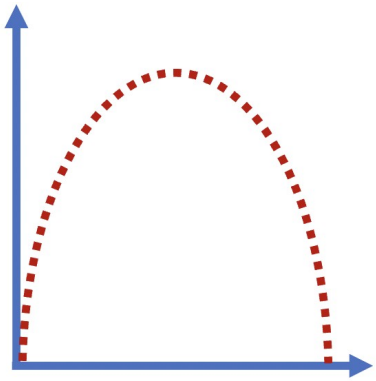
x	0	1	2	3	4	5	6	7	8
$g(x)$	0	240	448	624	768	880	960	1008	1024

Table 5. Function Table h

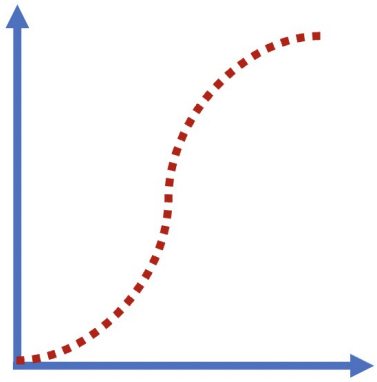
x	0	1	2	3	4	5	6	7	8
$h(x)$	0	23	91	191	309	425	521	582	599

Figure 3. Graph Sketches I, II, and III

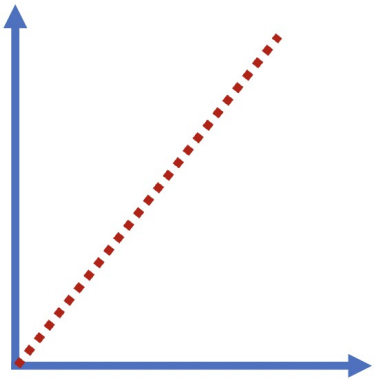
Sketch I:



Sketch II:



Sketch III:



Item 4. Directions

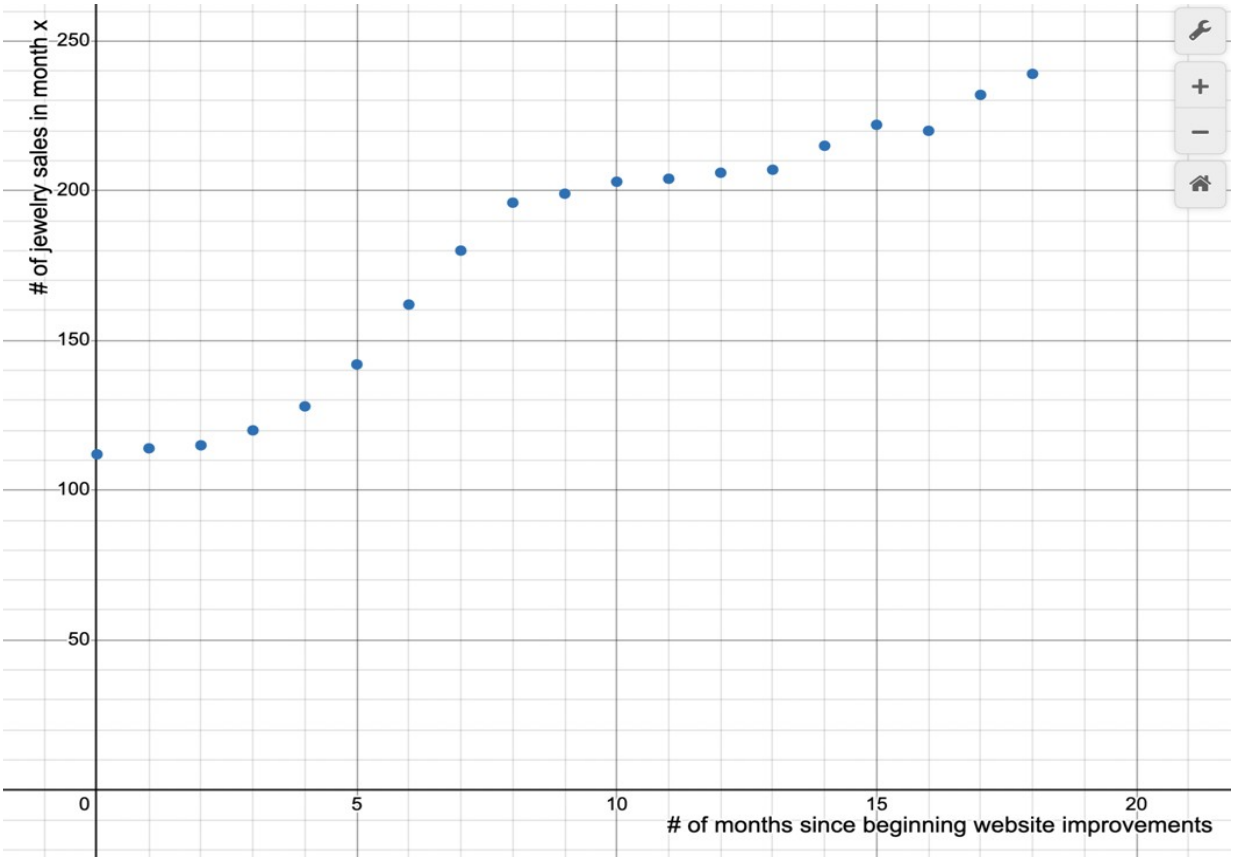
Item 4 has three tasks.

Ajana decided to try to boost her sales by improving her website. Making all the changes took her 18 months. She tracks her monthly sales to see how her website changes are affecting her sales. Below is the data Ajana collected during the 18 months she worked on improving the website.

Table 6. Months of Jewelry Sales

Number of months since starting website improvements	Number of jewelry sales
0	112
1	114
2	115
3	120
4	128
5	142
6	162
7	180
8	196
9	199
10	203
11	204
12	206
13	207
14	215
15	222
16	220
17	232
18	239

Figure 4. Months of Jewelry Sales



Item 4 Tasks

(A) Calculate the average rate of change of Ajana's sales over three or four specific intervals. How and why did you select those intervals?

(B) When were her sales increasing the fastest? Show and explain how you know using multiple representations such as the graph, numerical calculations, and spoken or written language.

(C) How certain do you think Ajana can be that these sales increases are related to her website work? What else might have caused her sales to increase?

PART 3. Fireworks

Modeling Real-World Phenomena with Quadratic Functions; Describing, Representing, and Interpreting Quadratic Models

A university is putting on a fireworks display to celebrate 100 years of operation. The fireworks rockets will be launched from the ground.

Item 1. Directions

Item 1 has five tasks.

The height in feet of a fireworks rocket launched from the ground is given by the function $f(t) = -16t^2 + 144t$, where t represents how much time in seconds has passed since the rocket was launched.

Item 1 Tasks

(A) Graph $f(t)$. Label the function and axes.

(B) Label the intercepts on your graph and explain what these points represent in this situation. (That is, what do these points tell you about the rocket's path and what is happening to it when?)

(C) What is the domain of $f(t)$ in general? What domain for $f(t)$ makes sense in this situation where $f(t)$ is modeling the height of the rocket over time? Explain why you think so.

(D) In the context of this problem (that is, only considering $f(t)$ over the domain that makes sense for the rocket's height), explain where $f(t)$ is increasing, decreasing, positive, and negative. For each, explain how you can tell.

(E) In the context of this problem (that is, only considering $f(t)$ over the domain that makes sense for the rocket's height), what are the minima and maxima of $f(t)$? For each, explain how you can tell and what each represents in the context of the problem.

Item 2. Directions

Item 2 has two tasks.

The next year, the celebration team makes some adjustments to the fireworks show. After making some changes, they find that the relationship between how many seconds have passed (represented by t) and the height of the rocket at time t (represented by $f(t)$) can be modeled by the function $f(t) = -16t^2 + 144t + 160$.

Item 2 Tasks

(A) Show how you can use factoring and the Zero Product Property to figure out how long it will take for the rocket to hit the ground again after it is launched, when it will hit its highest point, and how high it will be at its highest point. Use a graph of the function to explain how you know your answers are correct.

(B) One of the changes the team made this year was launching the fireworks rocket from a raised platform rather than from the ground. How high is the podium? Show and explain how you figured it out using the equation **and** the graph.

Item 3. Directions

Item 3 has one task.

The following year, the celebration team is deciding whether they will use the same launch setup they used last year (where the height of the fireworks at time t is modeled by the function $f(t)=-16t^2+144t+160$), or whether they will use a new, slightly different setup instead, which can be modeled by the function $g(t)$ in table 7 below.

Item 3 Task

Compare the two different models that result from the two different launch setups:

- original setup: $f(t)=-16t^2+144t+160$
- new setup: $g(t)$ (represented by table 7)

Which setup launches the fireworks from a higher platform? Which sends the fireworks higher at the maximum point? With which setup will the fireworks debris take longer to hit the ground?

Use graphs, tables, and equations to show and explain how you found your answers.

Table 7. Table Representation of $g(t)$

t	$g(t)$
-5	-1224
-4	-896
-3	-600
-2	-336
-1	-104
0	96
1	264
2	400
3	504
4	576
5	616
6	624
7	600
8	544
9	456
10	336
11	184
12	0
13	-216
14	-464
15	-744